Quantum walks

Daniel J. Bernstein
University of Illinois at Chicago

Focusing on quantum walks
as an algorithm-design tool:
e.g. Grover's algorithm.
e.g. Ambainis's algorithm.

Can also study quantum walks
on much more general graphs.
2008 Childs, 2009 Lovett–Cooper–Everitt–Trevers–Kendon:
Can view, e.g., Shor's algorithm
as quantum walk on Shor graph.

Examples of applications to crypto

Minimum asymptotic ops known,
assuming plausible heuristics:

pre-q	post-q	problem
1	0.5	cipher
ρ	$\rho/2$	McEliece
$0.791\ldots$	$0.462\ldots$	MQ
$0.290\ldots$	$0.241\ldots$	subset sum

"Pre-q" e: as $n \to \infty$, $2^{(e+o(1))n}$
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Step 2: "Grover diffusion".
Negate a around its average.
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Step 2: "Grover diffusion". Negate a around its average. This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Grover's algorithm

Assume: unique $s \in \{0,1\}^n$ has $f(s) = 0$.

Traditional algorithm to find s: compute f for many inputs, hope to find output 0. Success probability is very low until #inputs approaches 2^n.

Grover's algorithm takes only $2^{n/2}$ reversible computations of f. Typically: reversibility overhead is small enough that this easily wins for all sufficiently large n.

Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion". Negate a around its average. This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits. With high probability this finds s.

algorithm

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Negate a around its average.

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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.

With high probability this finds s.

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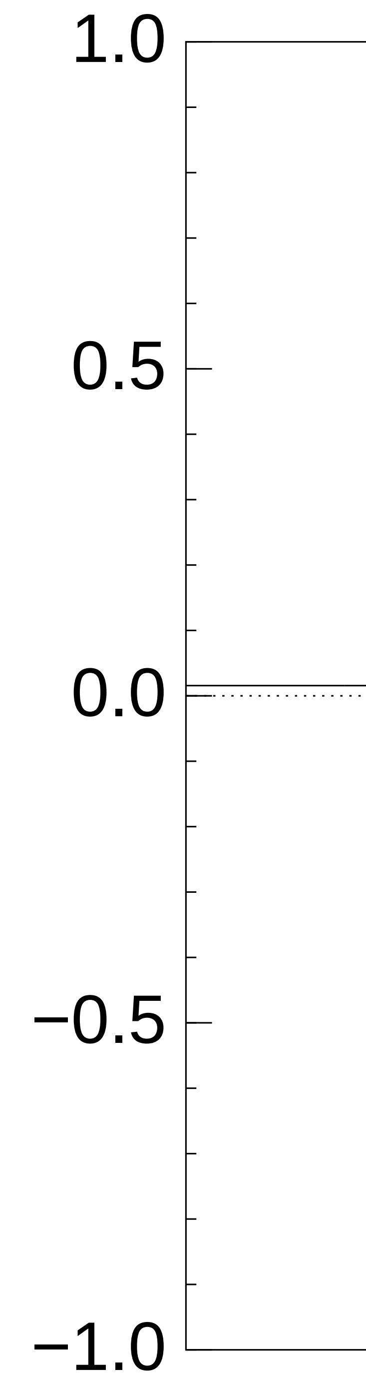

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about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph
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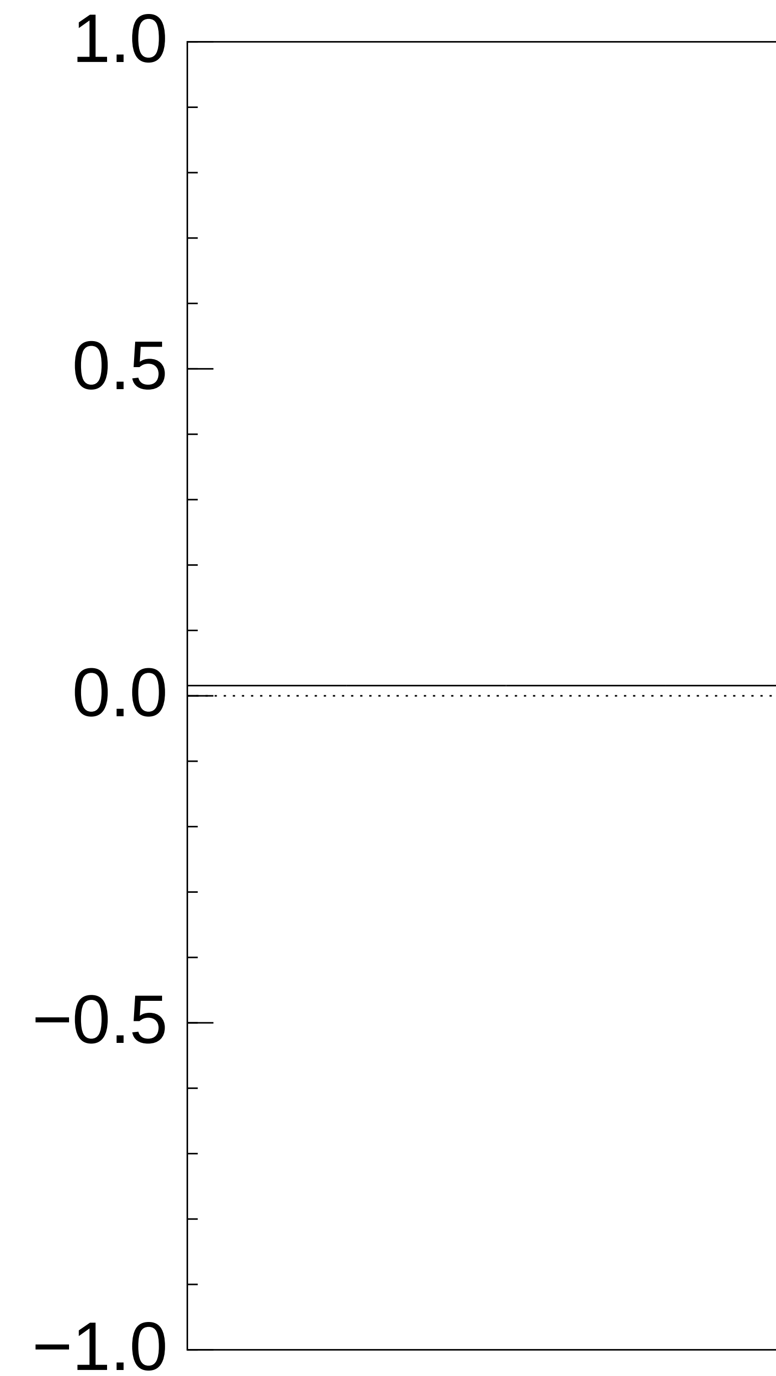

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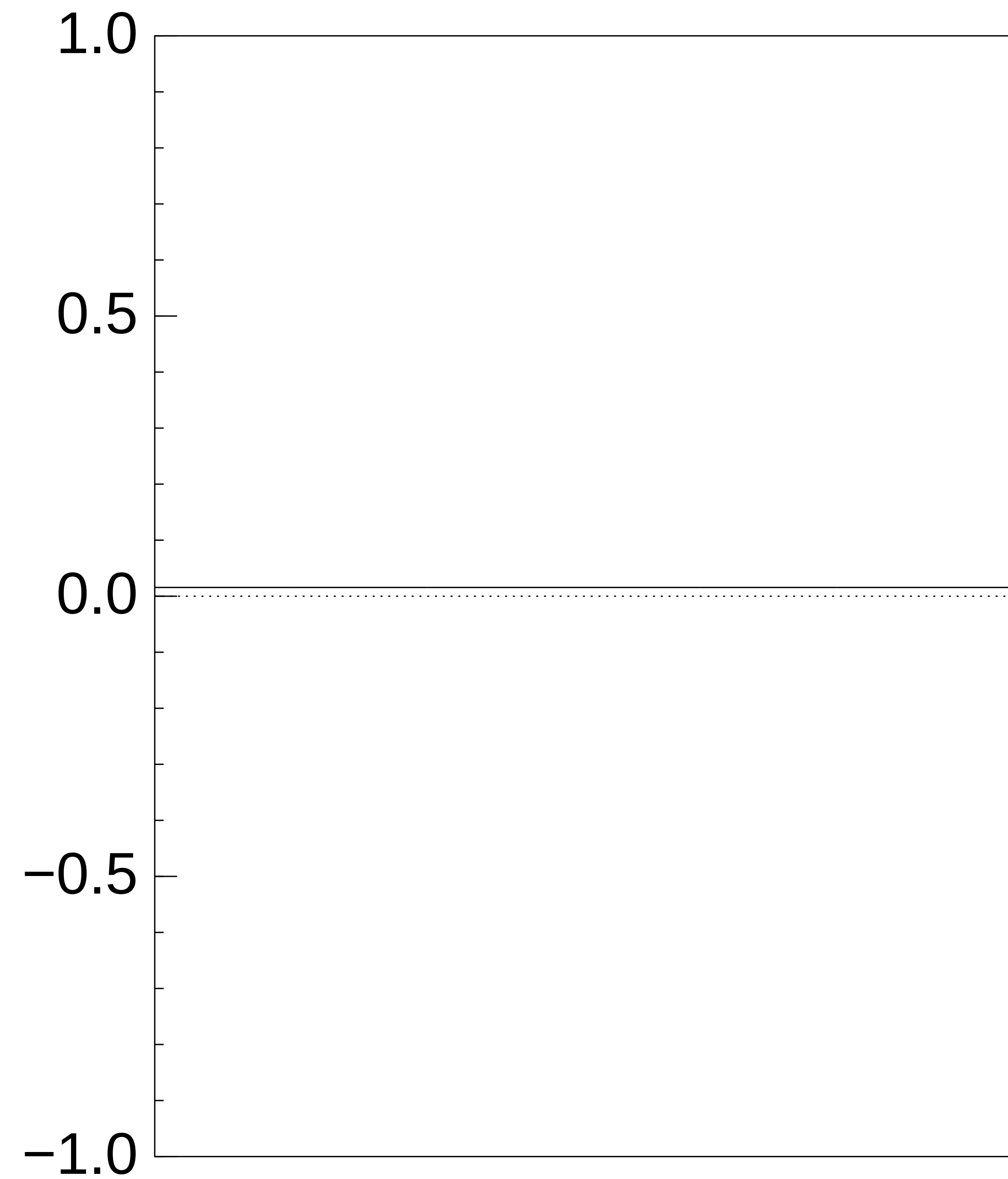
Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after 0 steps:



Start from uniform superposition
a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
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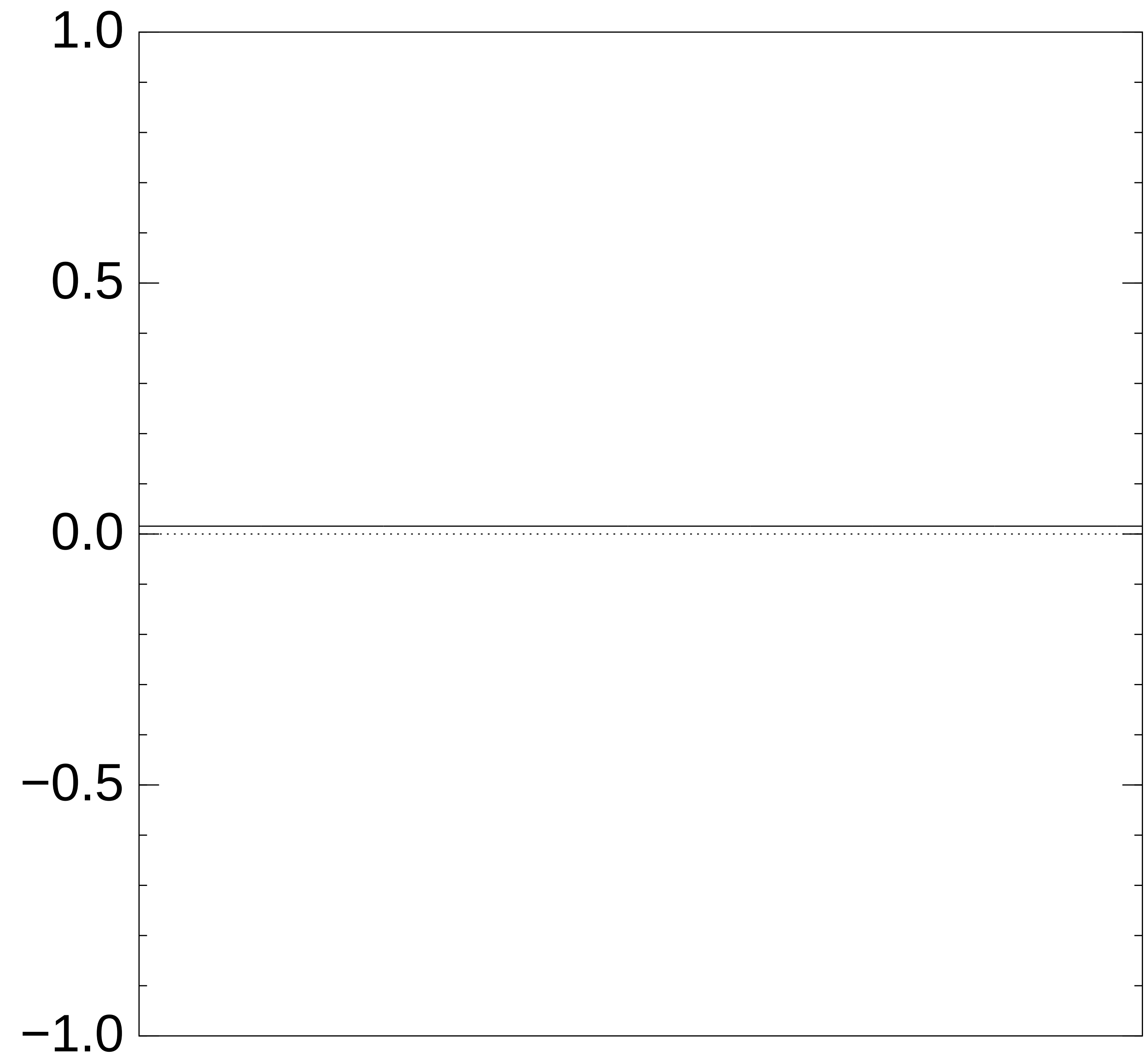
Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after 0 steps:



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
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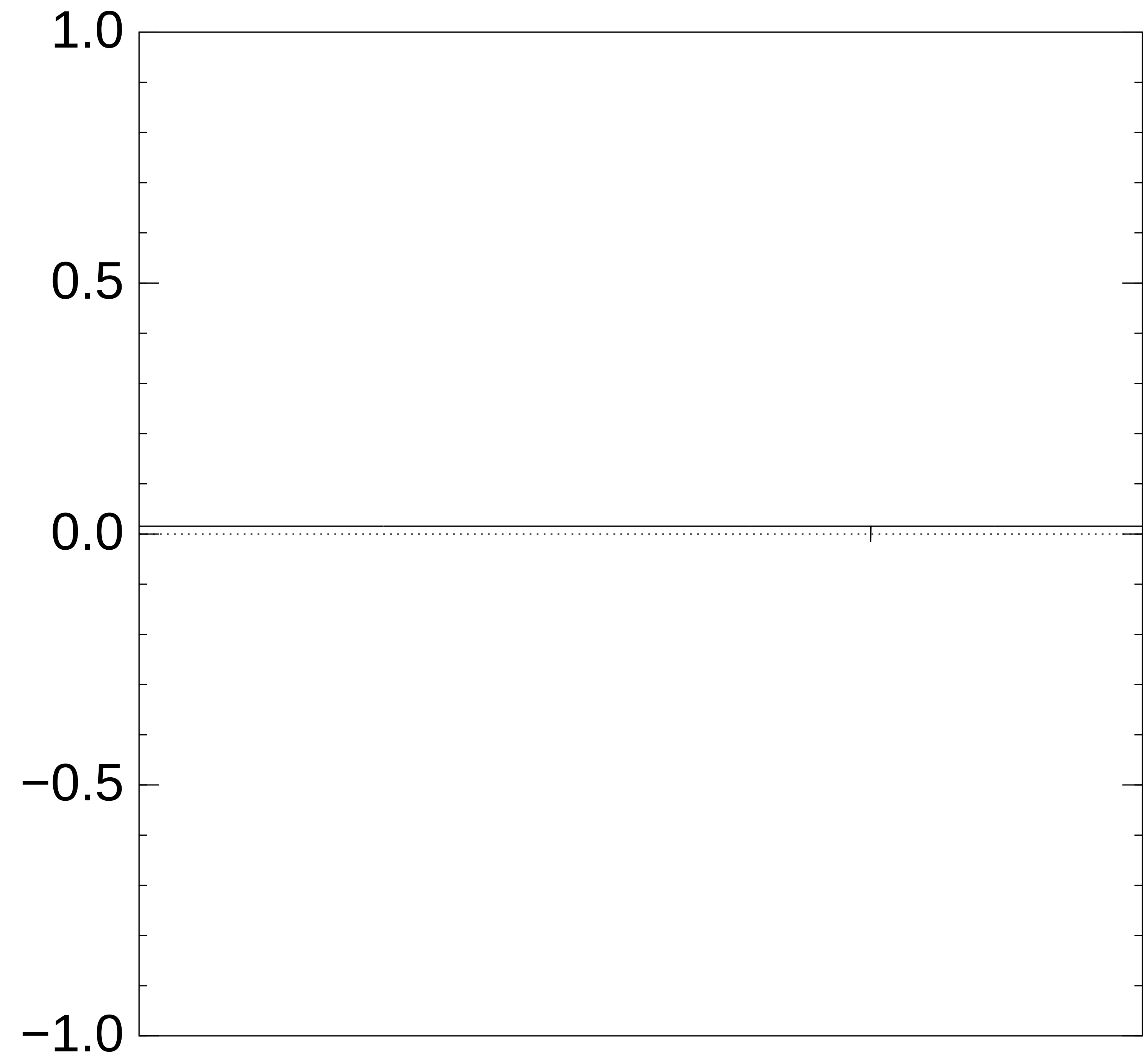
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1:



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

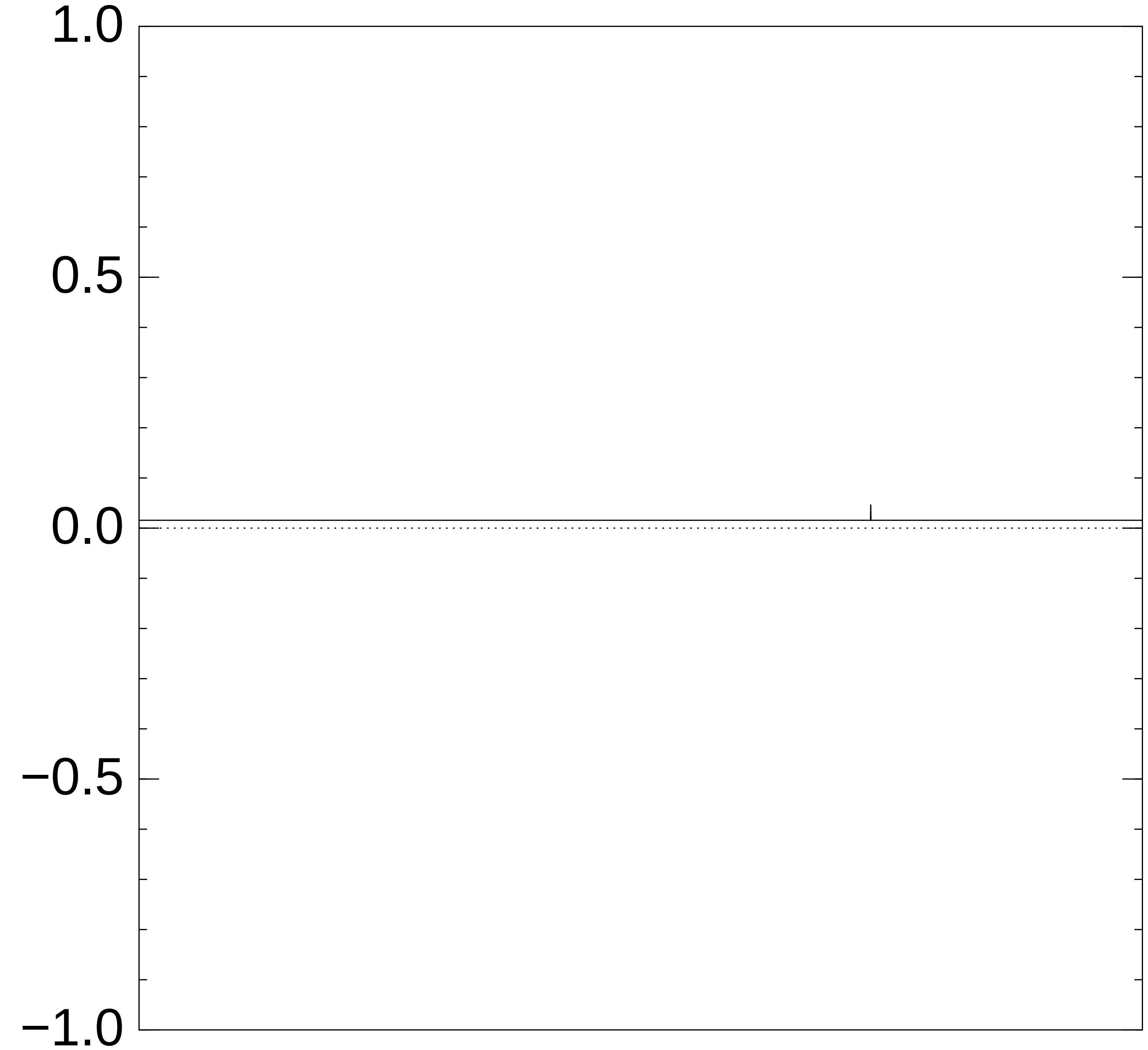
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2:



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

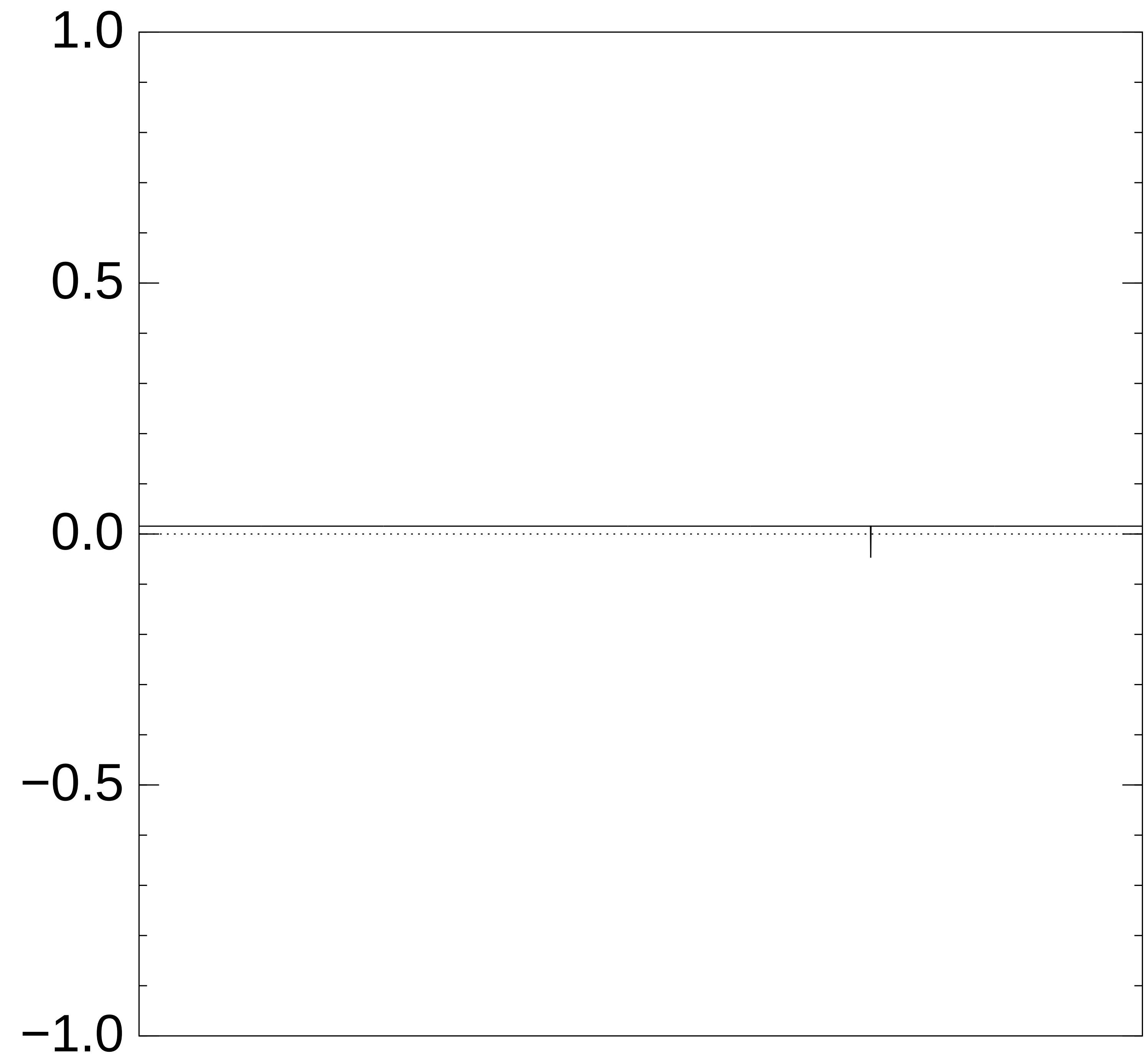
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2 + Step 1:



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

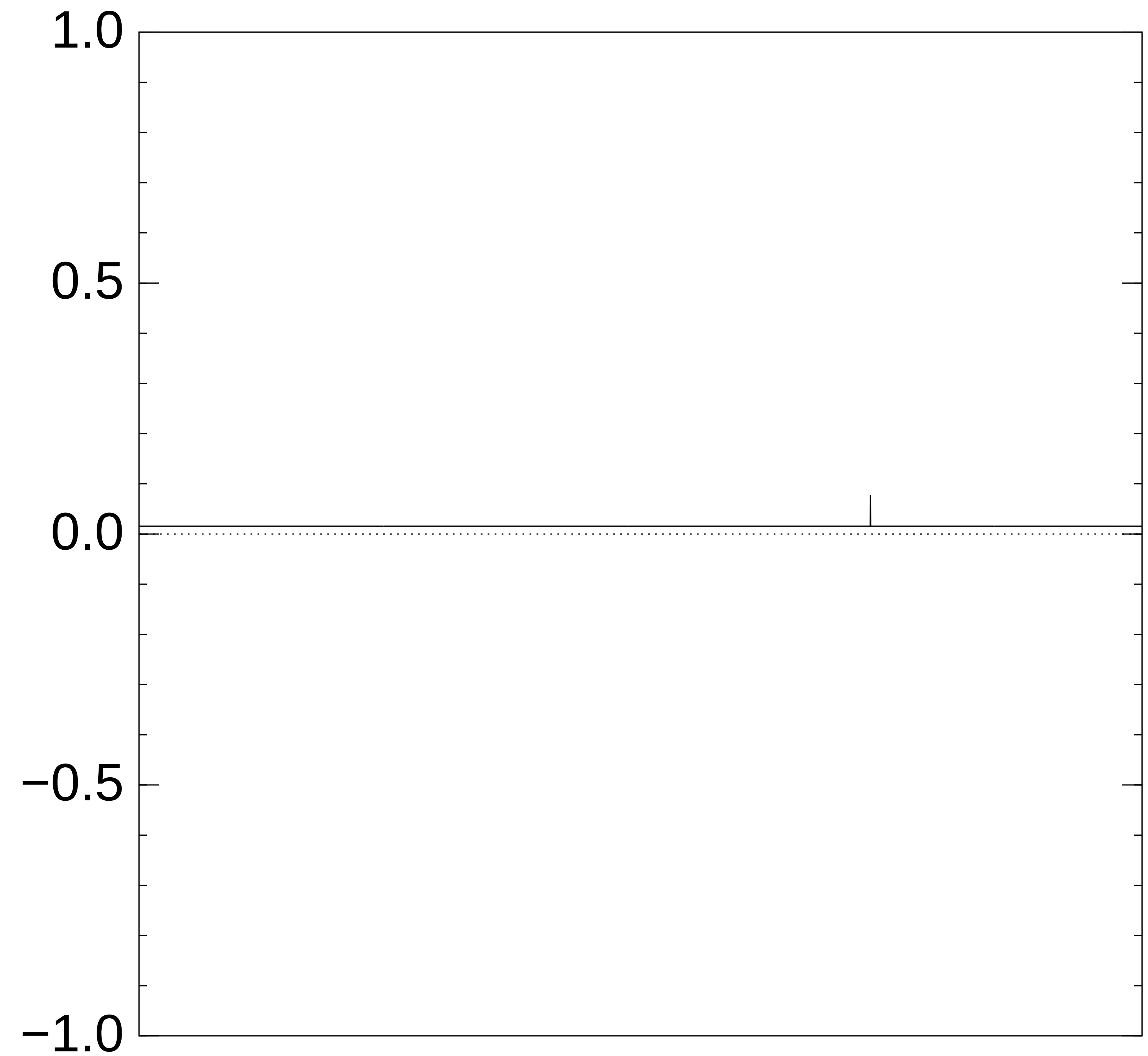
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $2 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

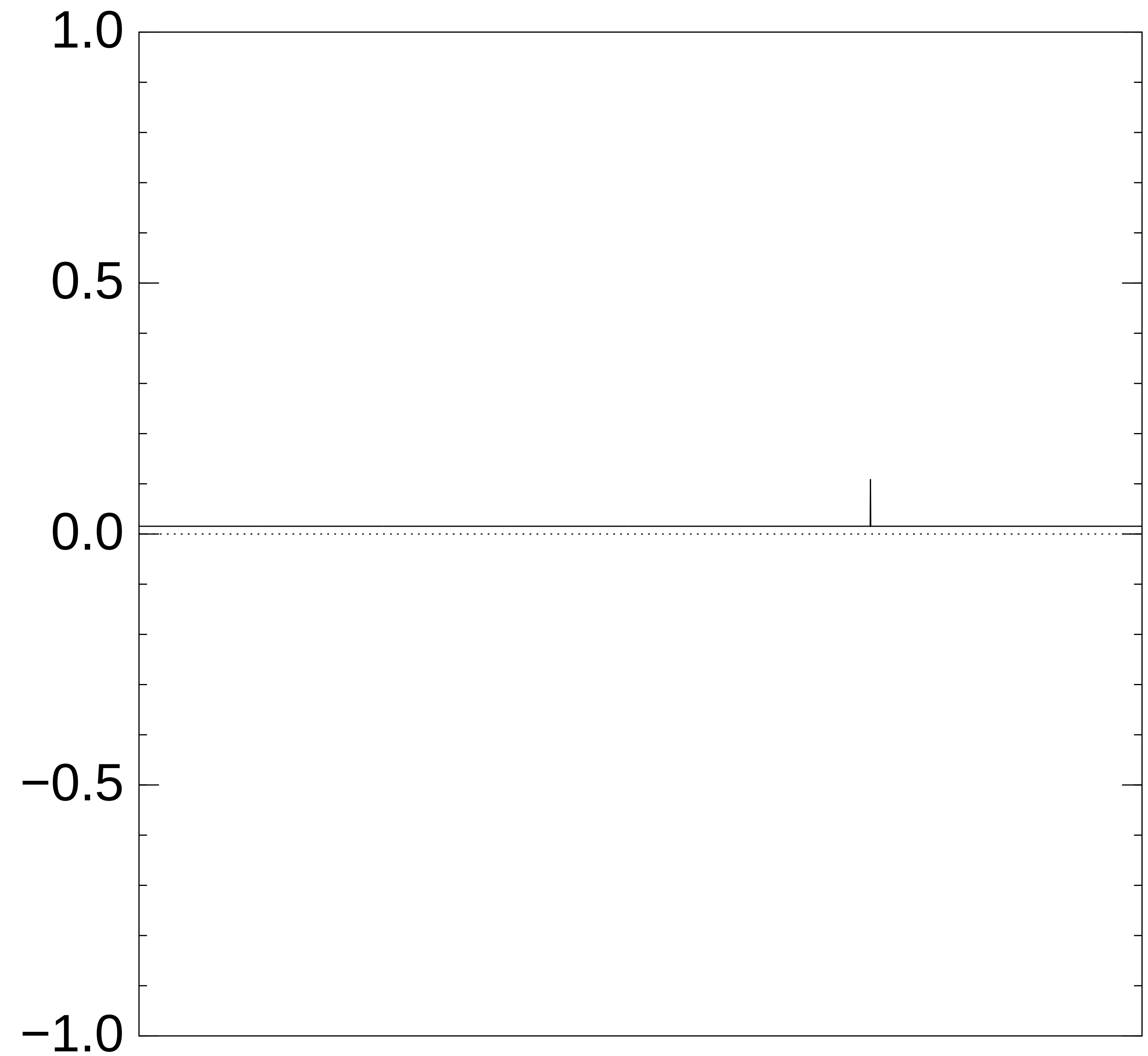
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $3 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

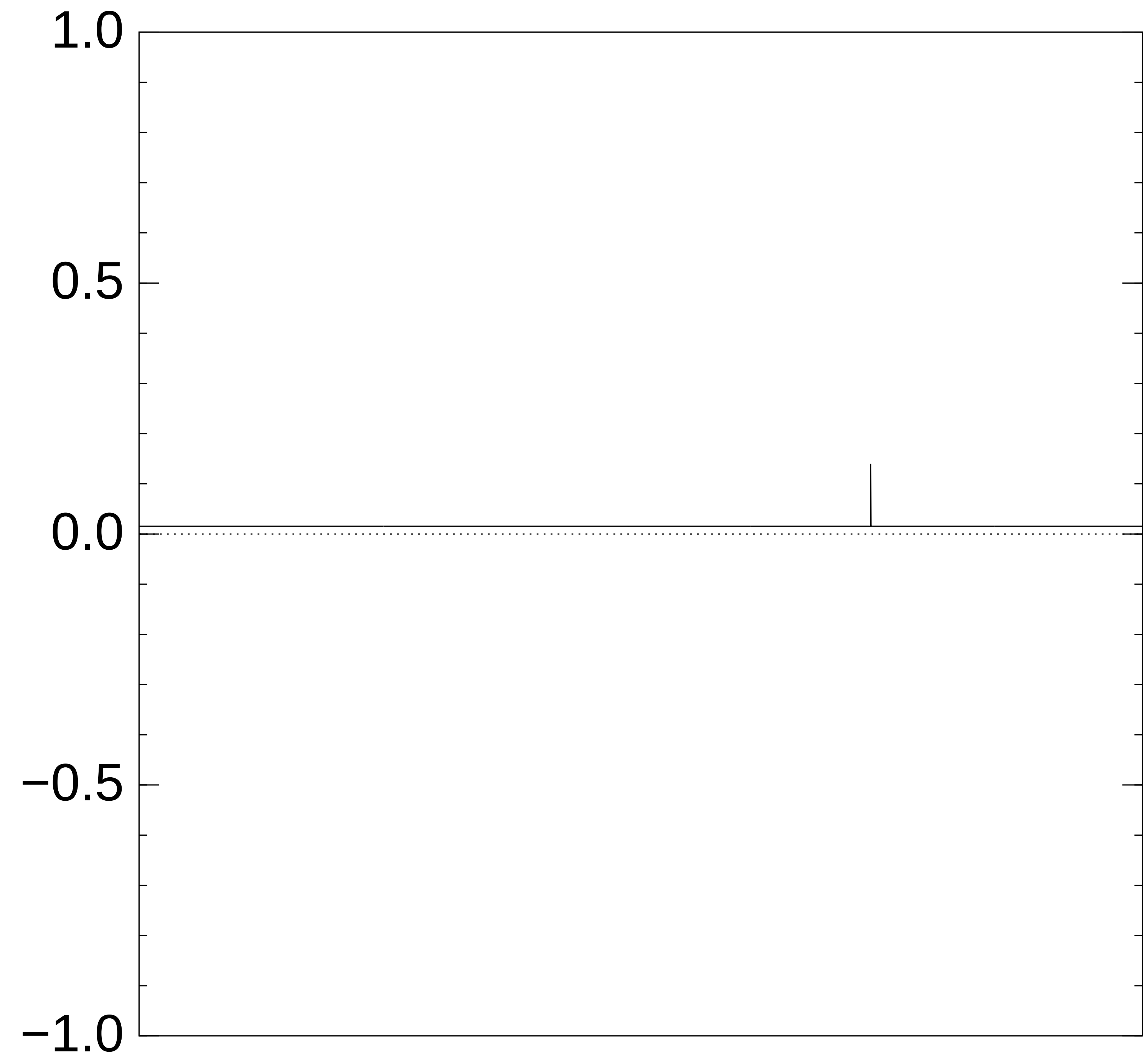
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $4 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

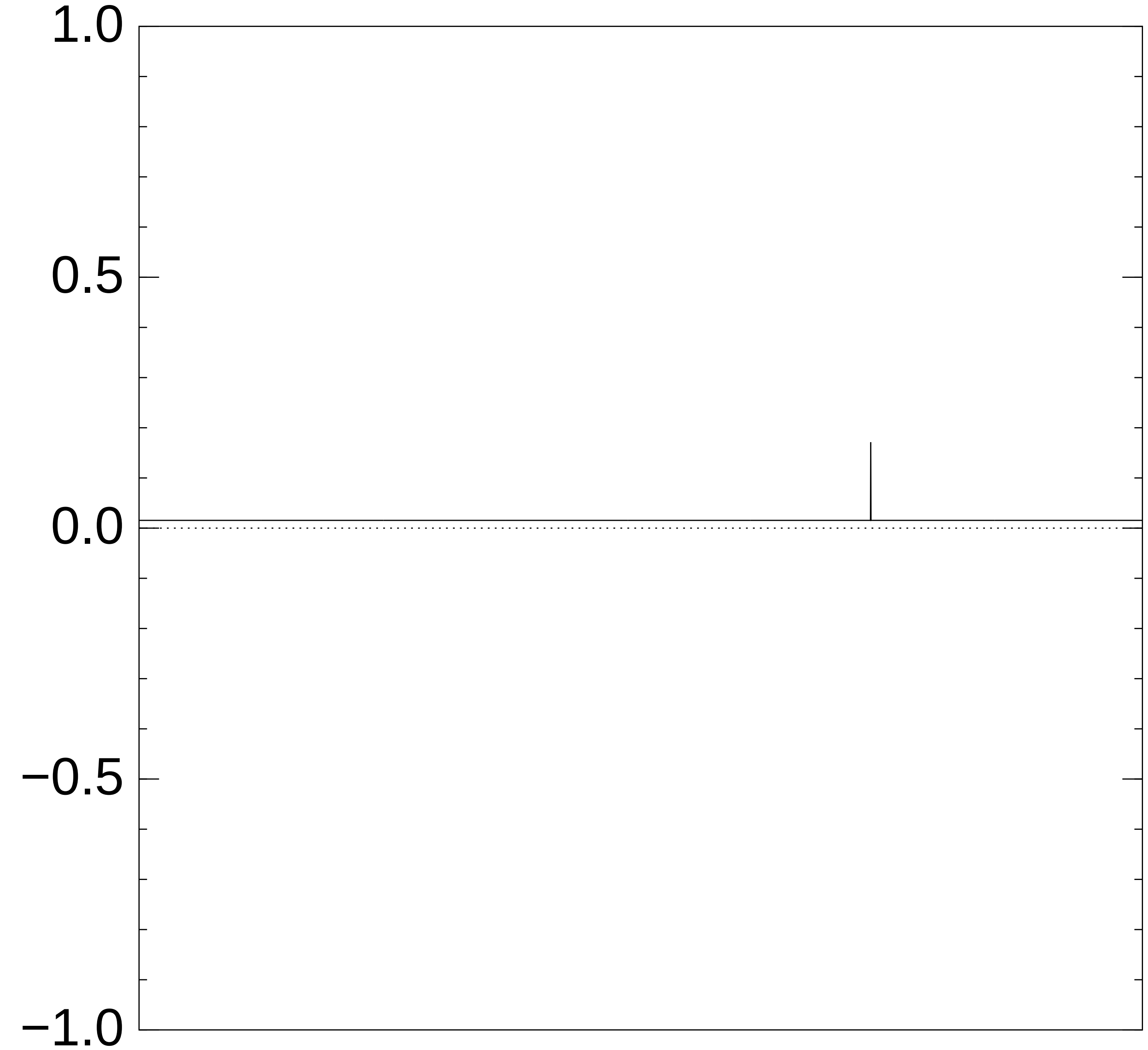
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

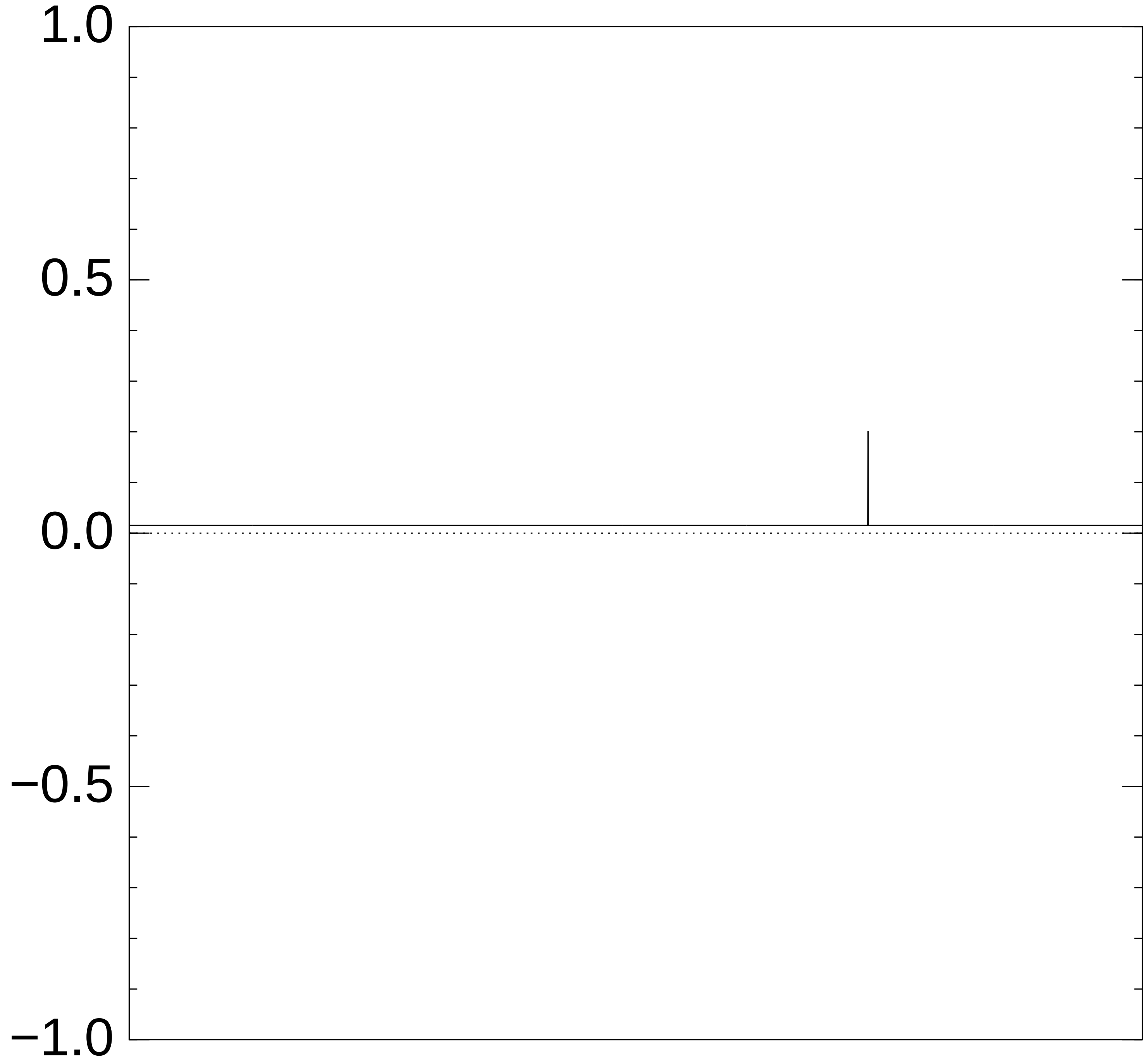
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $6 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

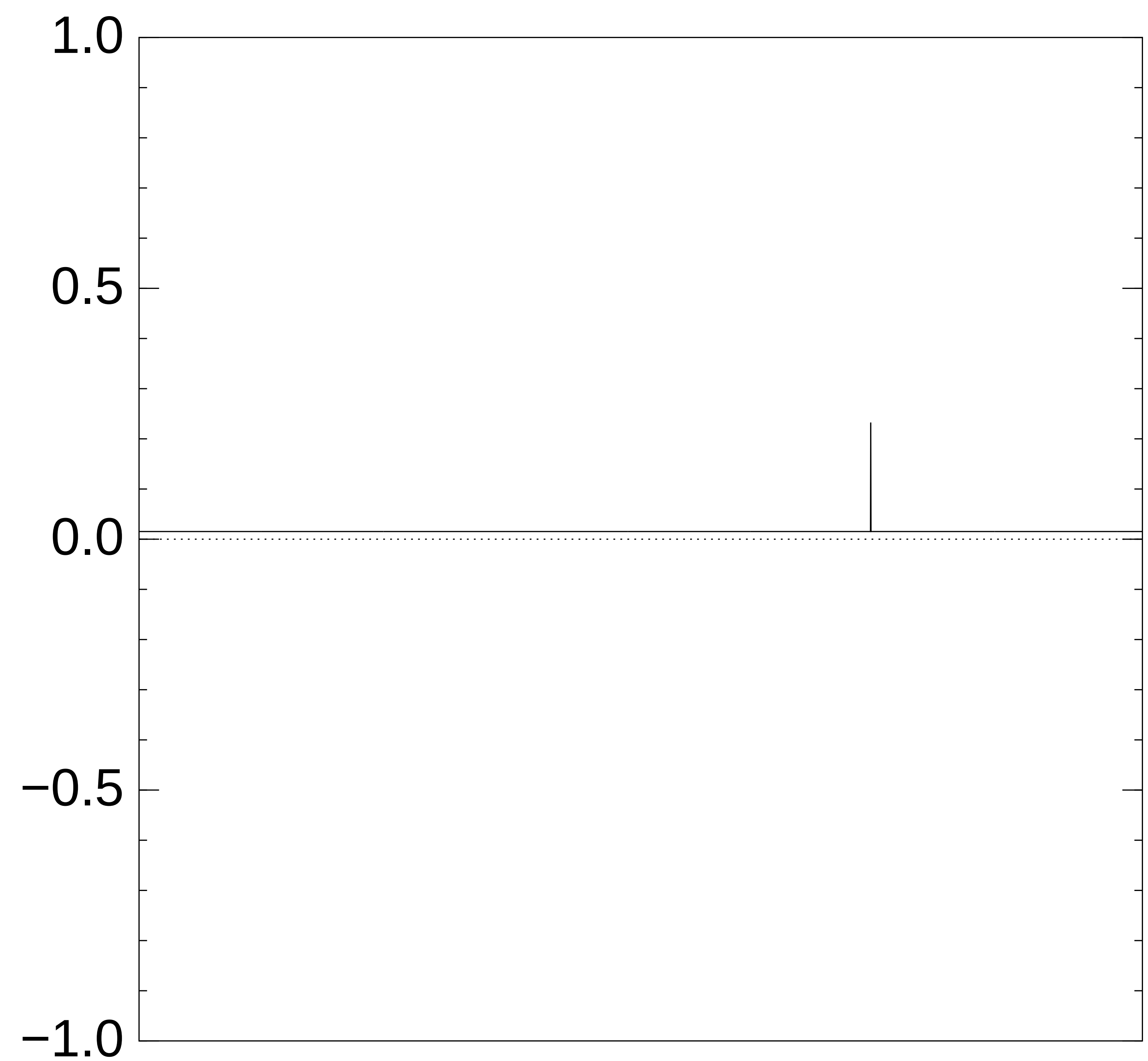
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $7 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

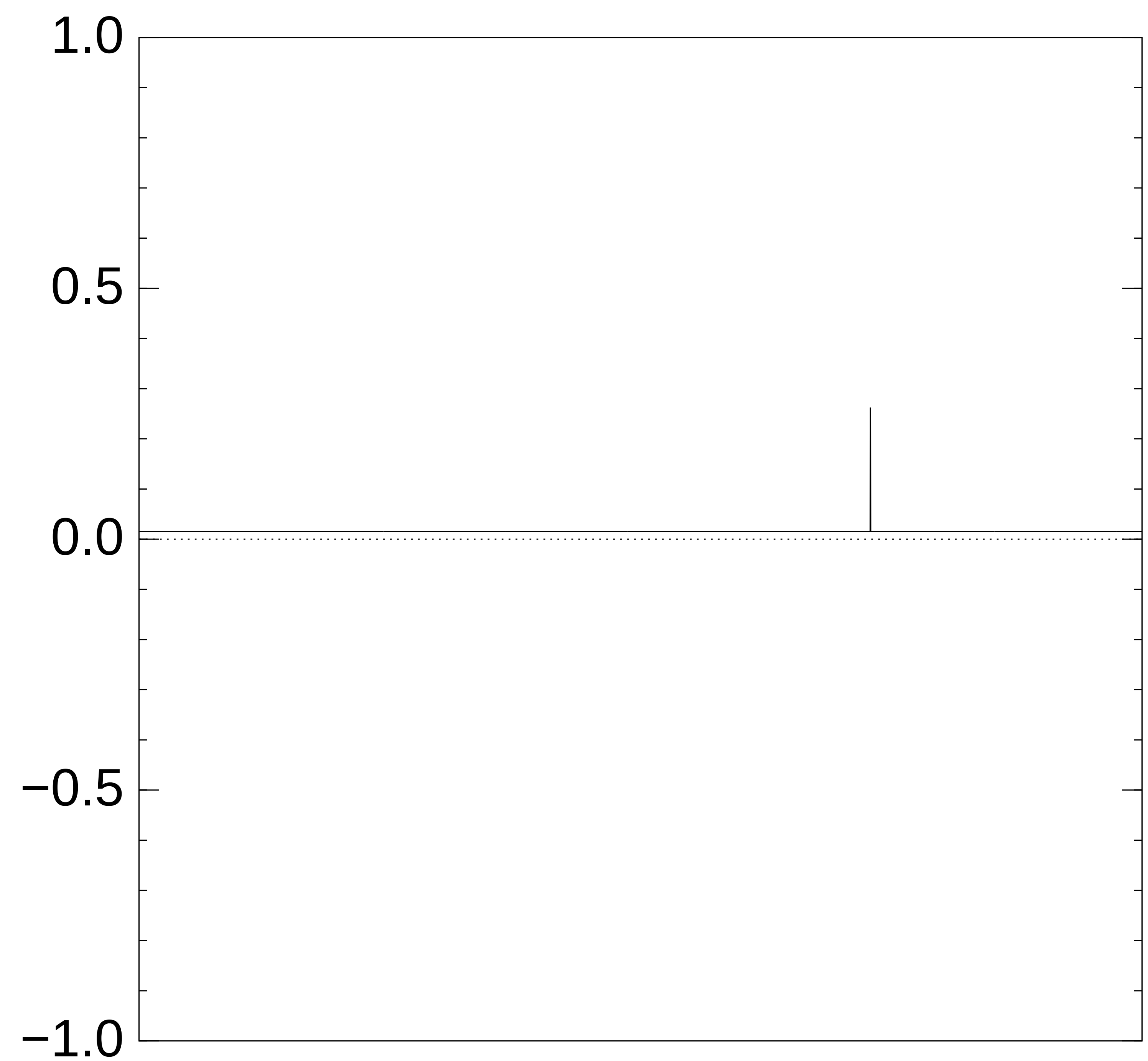
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

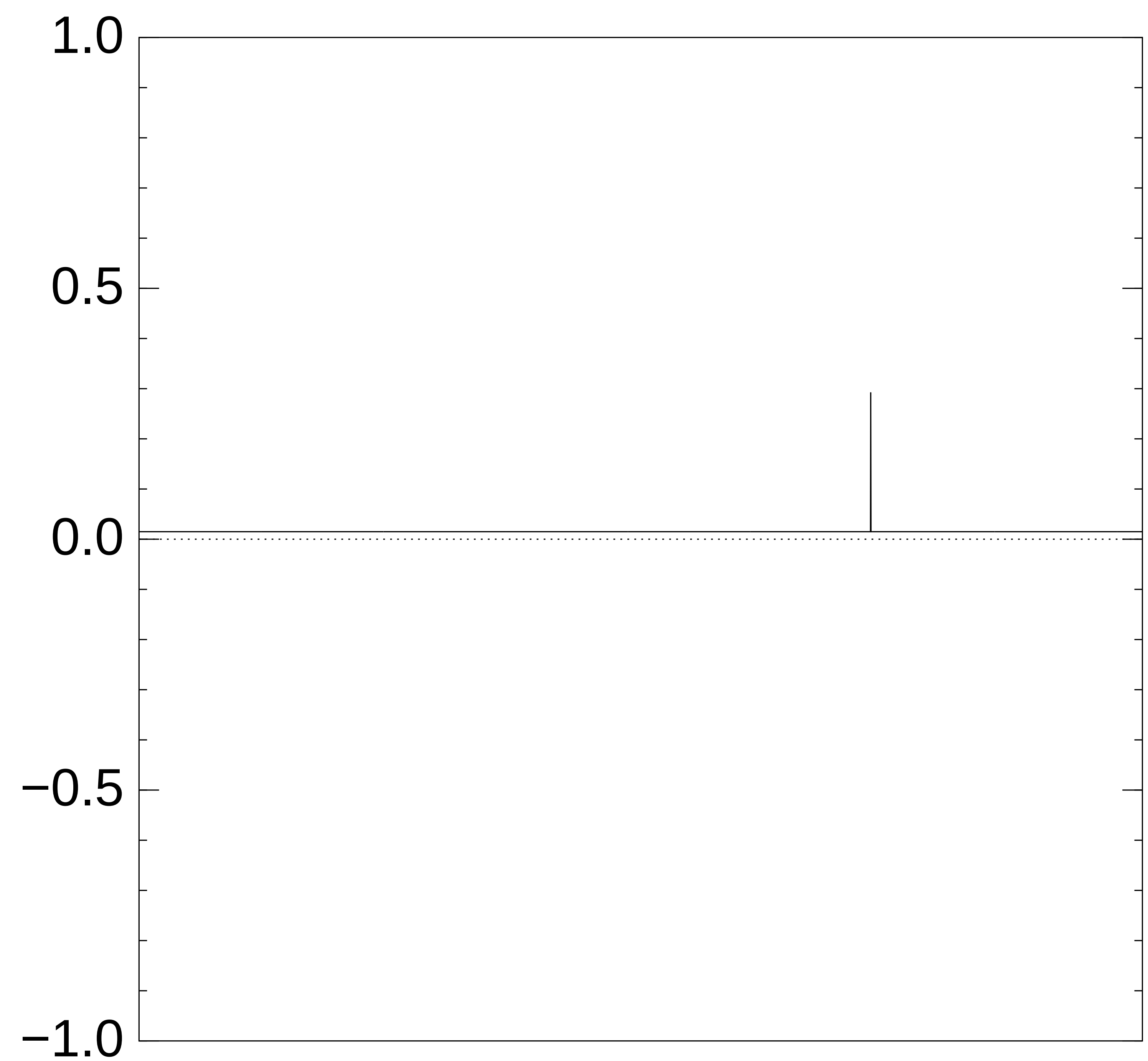
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $9 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

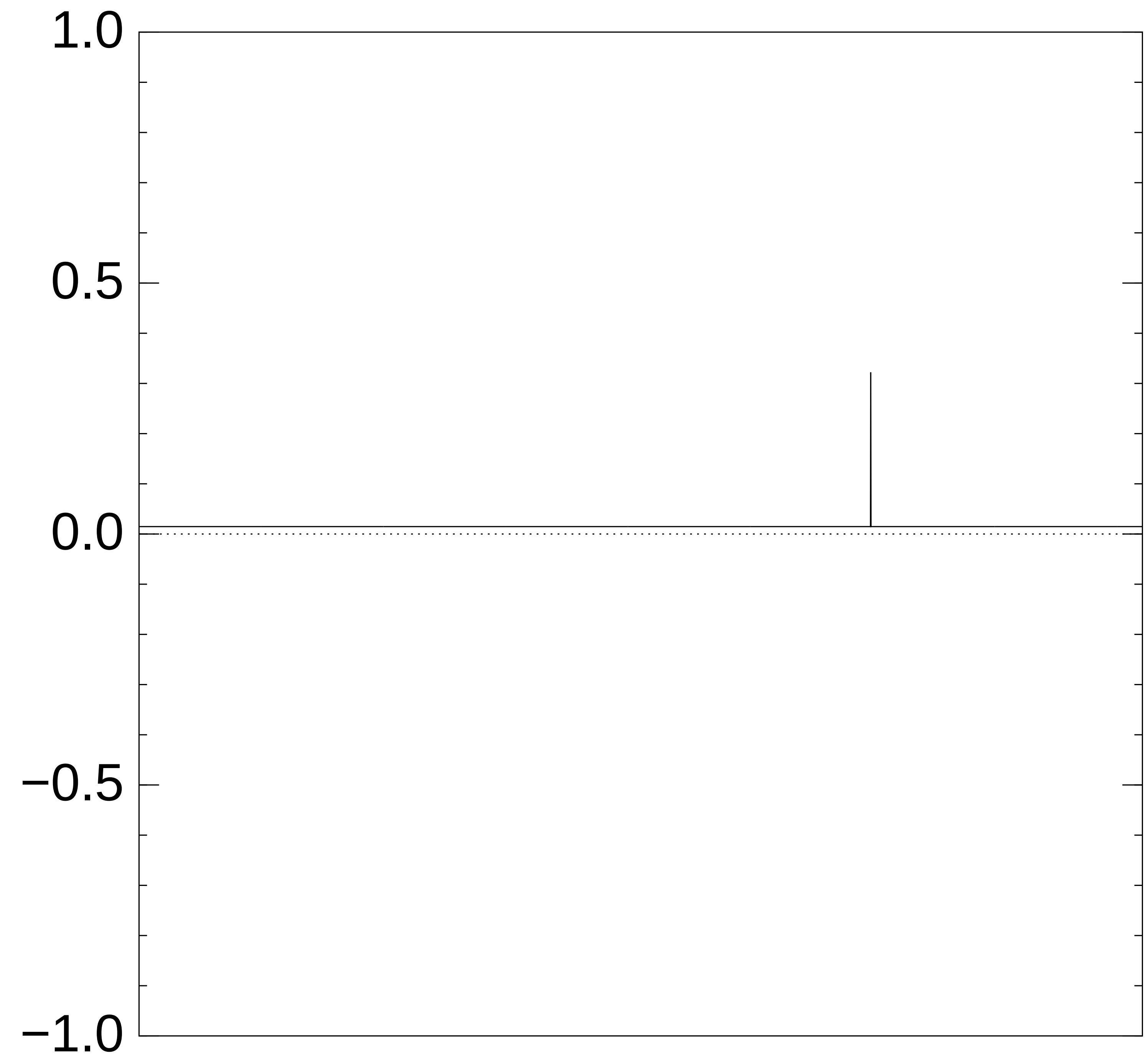
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

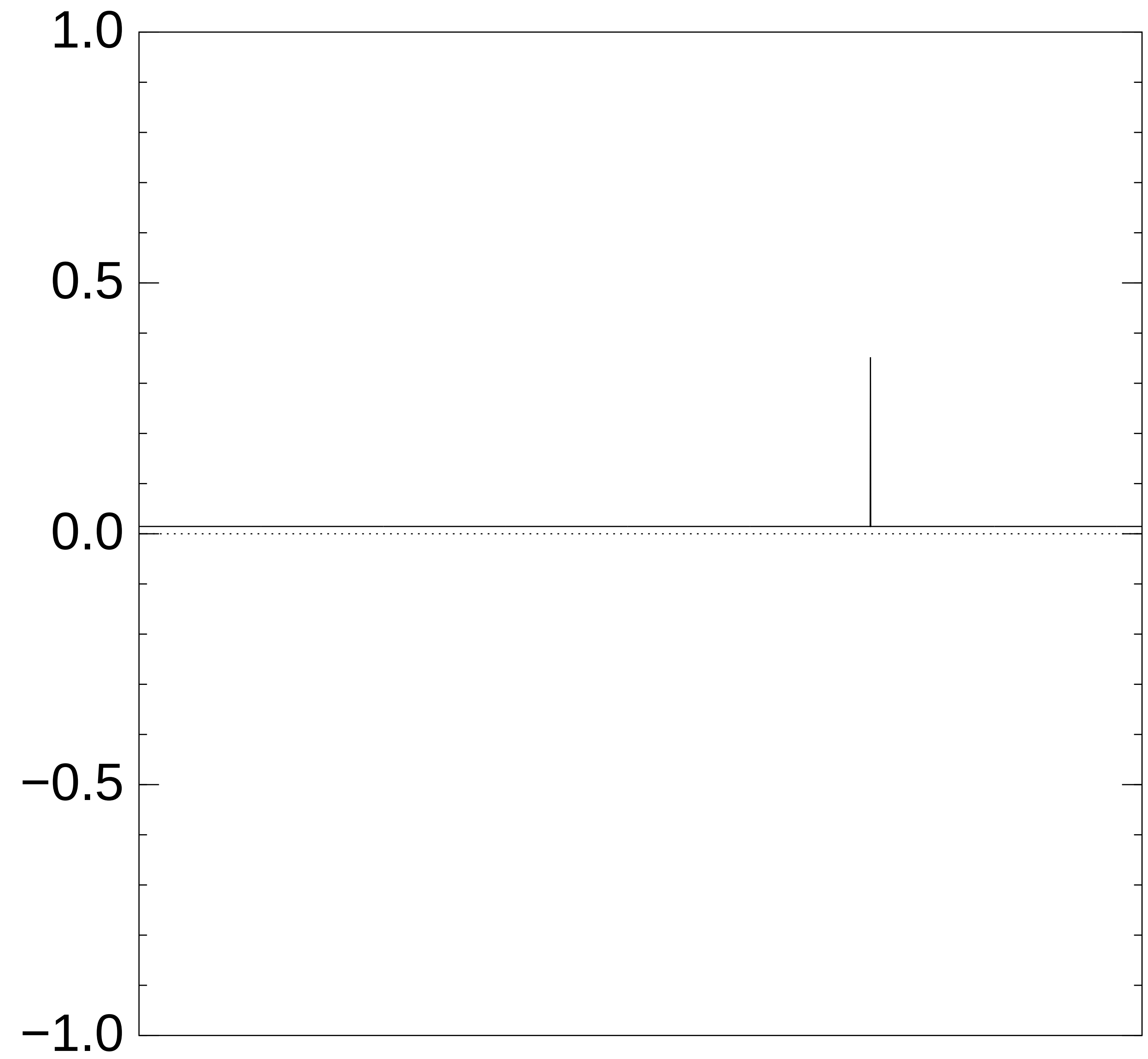
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

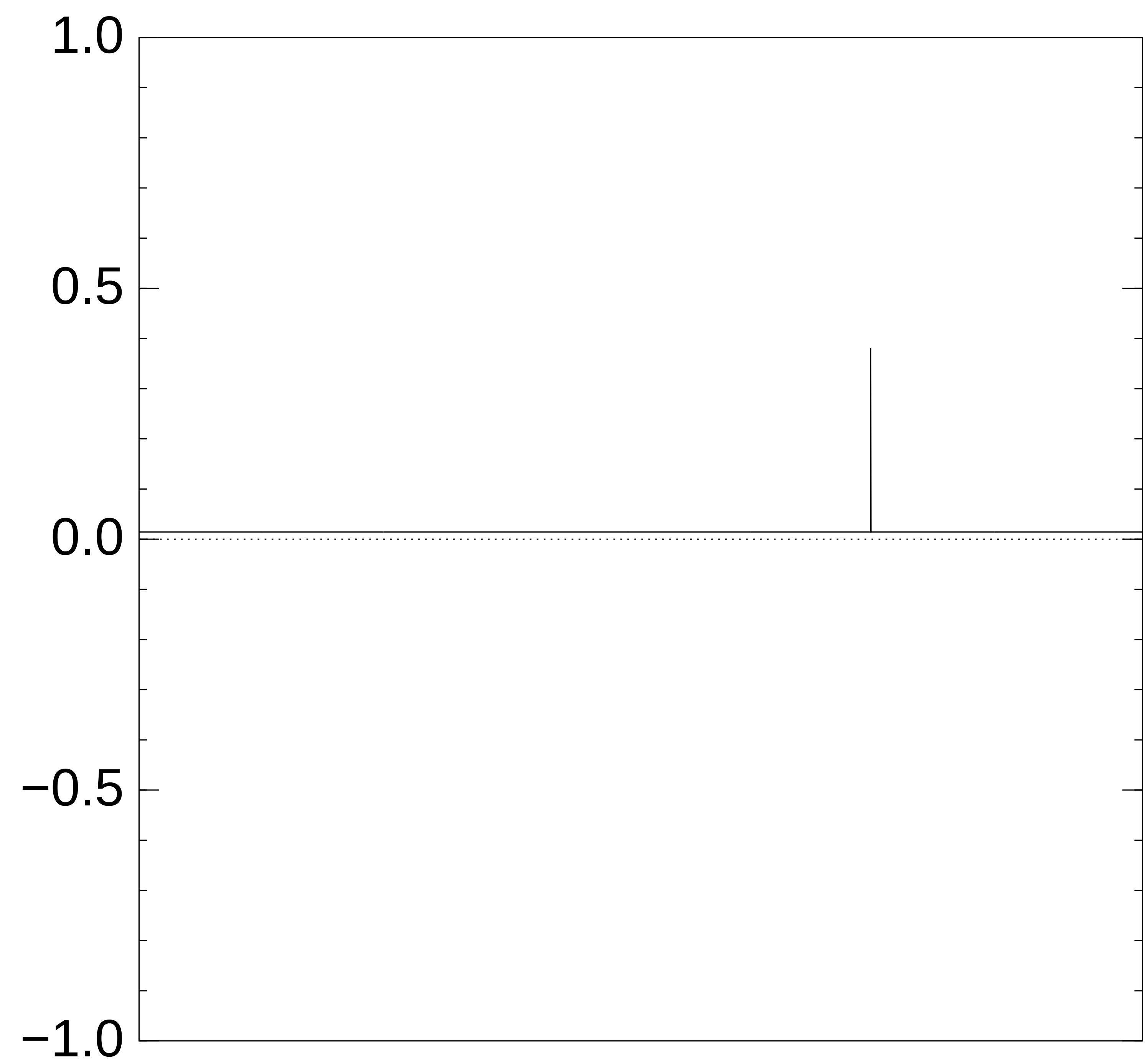
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $12 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

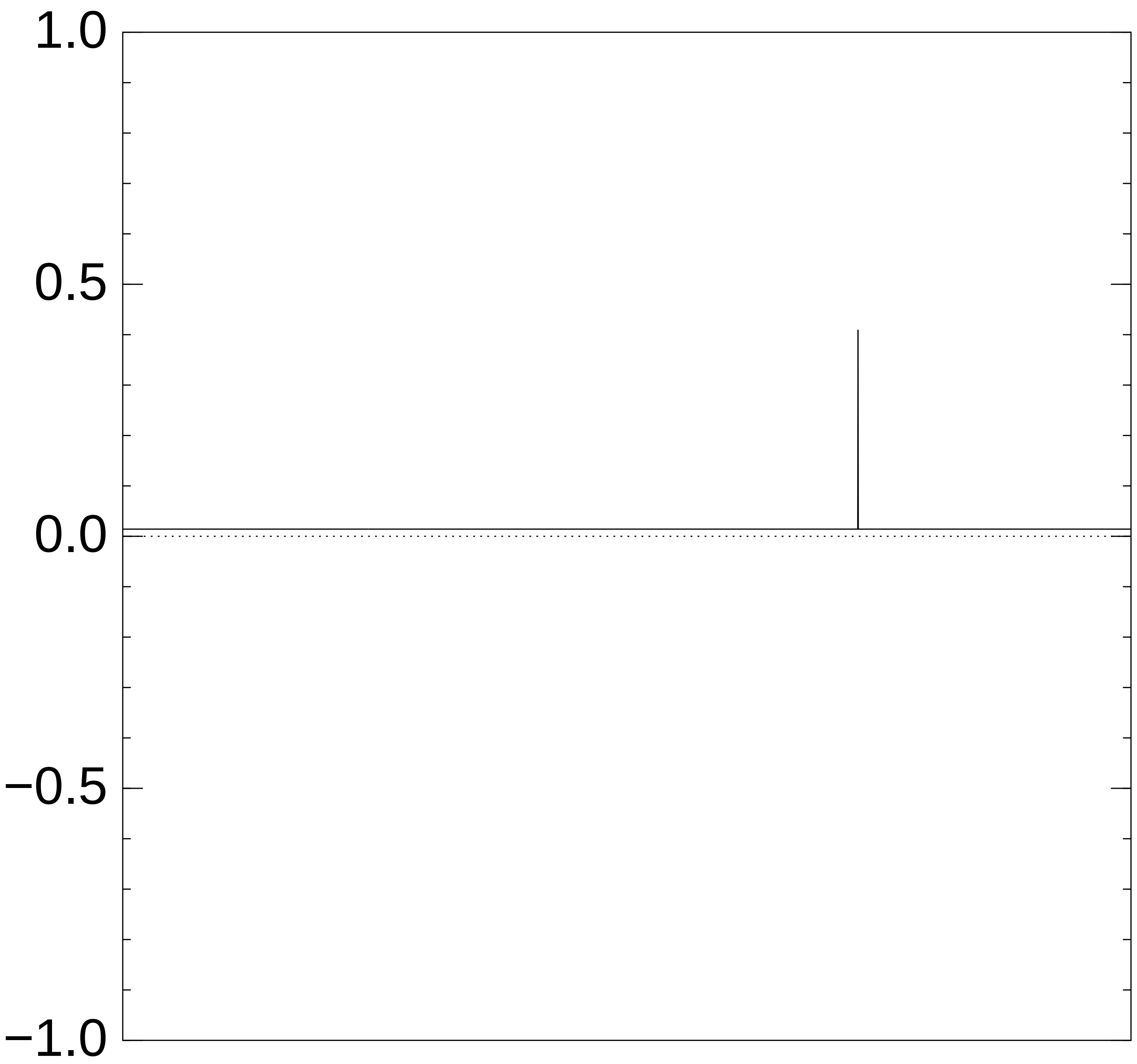
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $13 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

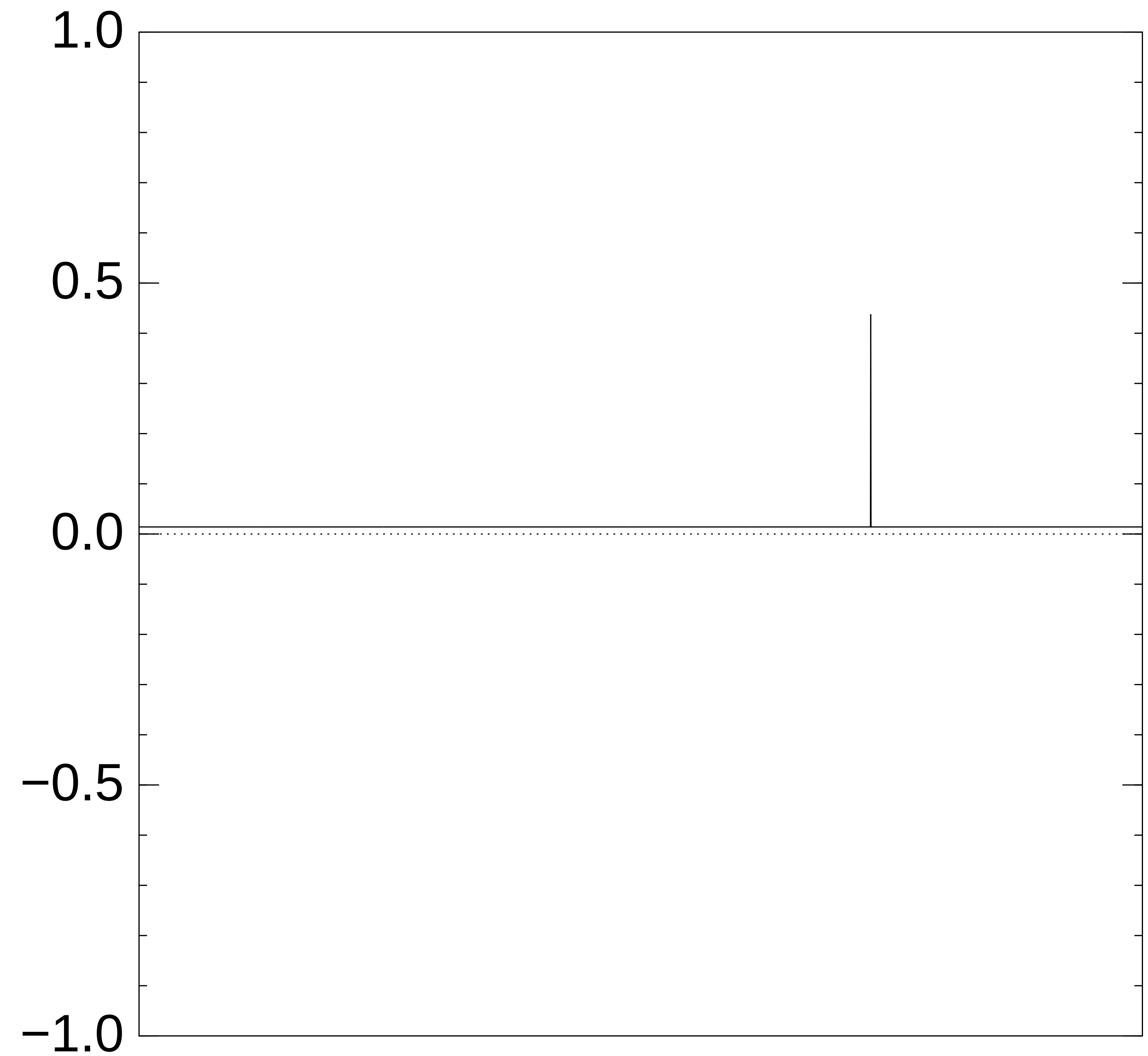
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $14 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

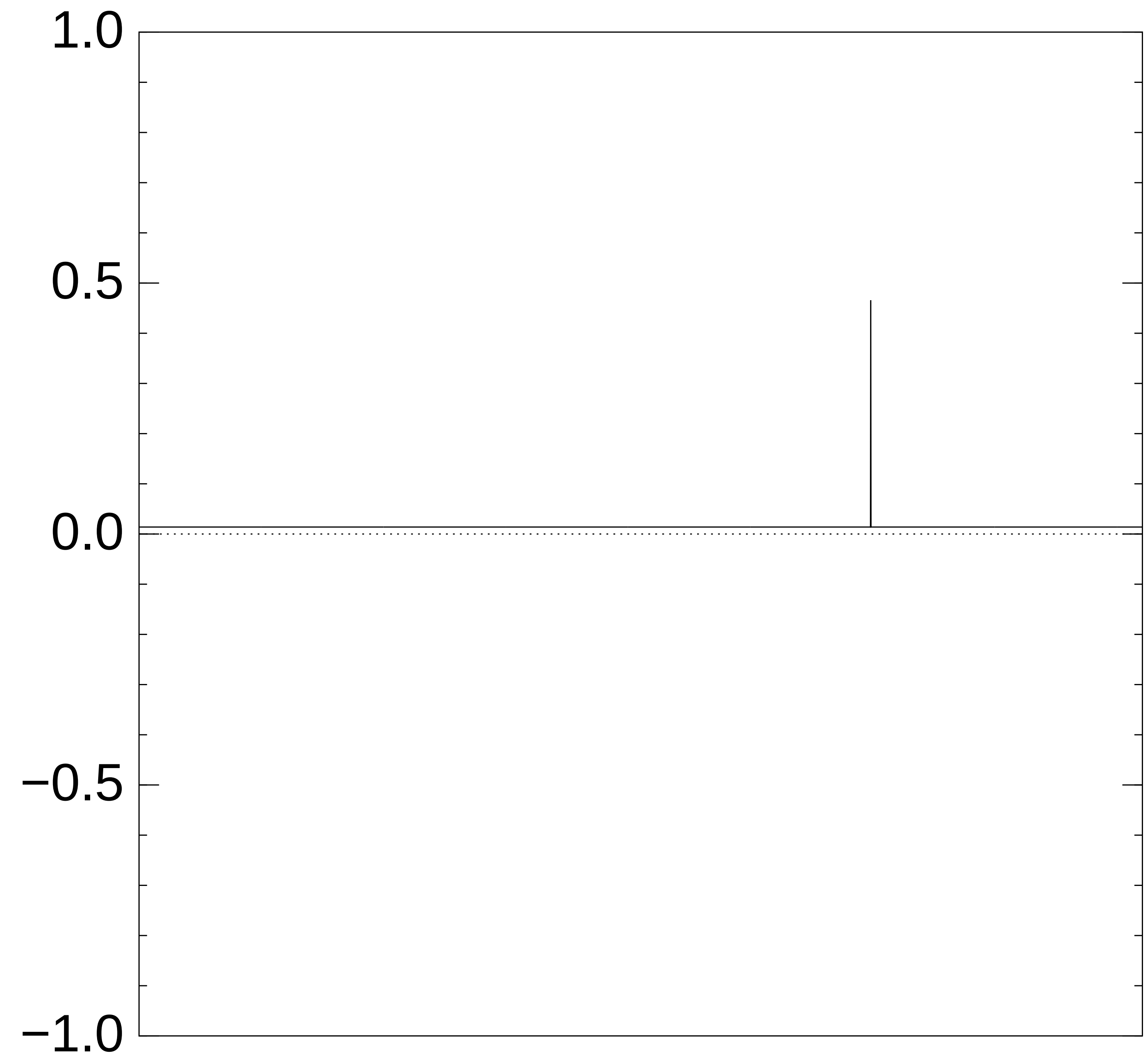
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $15 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

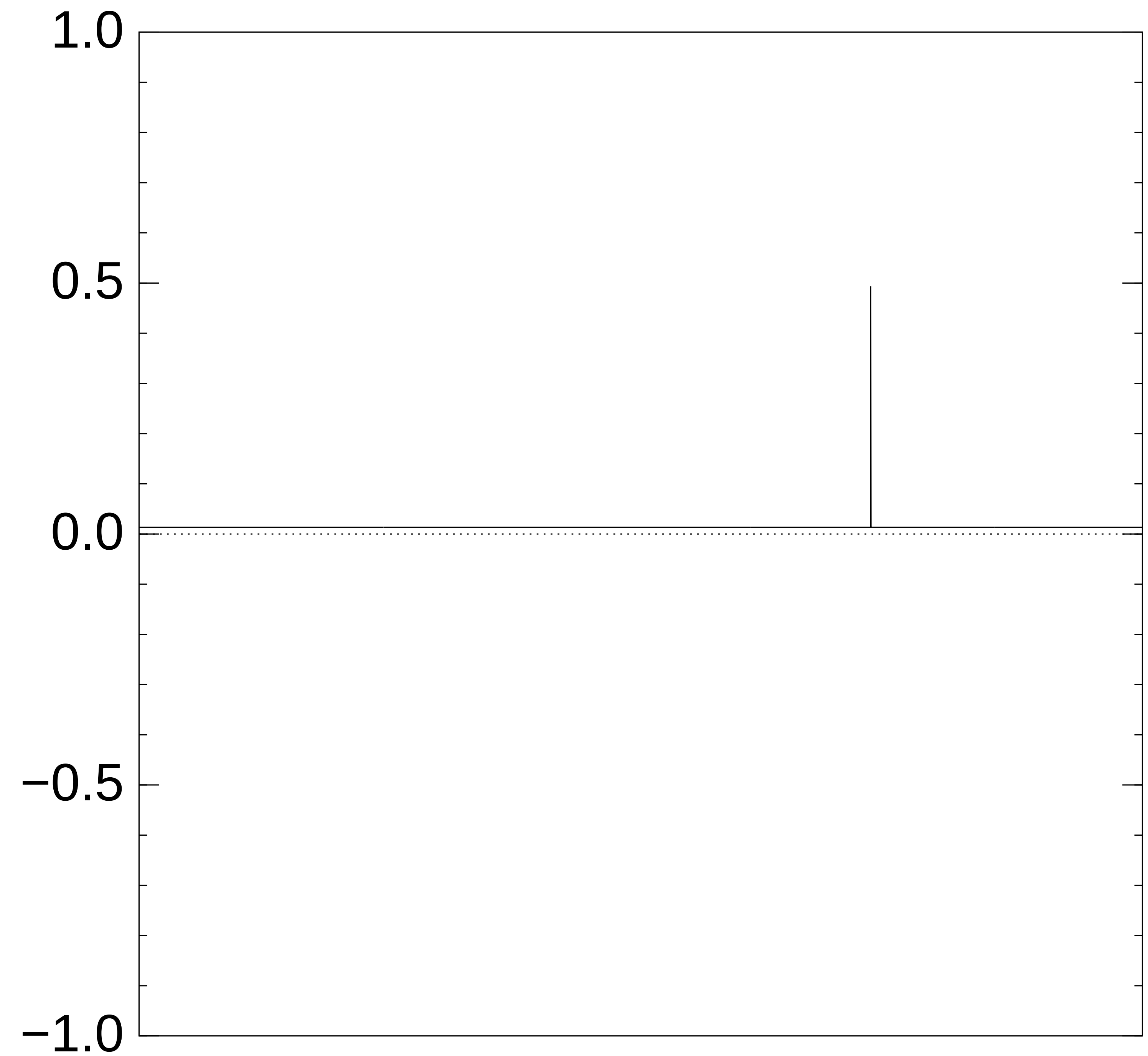
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $16 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

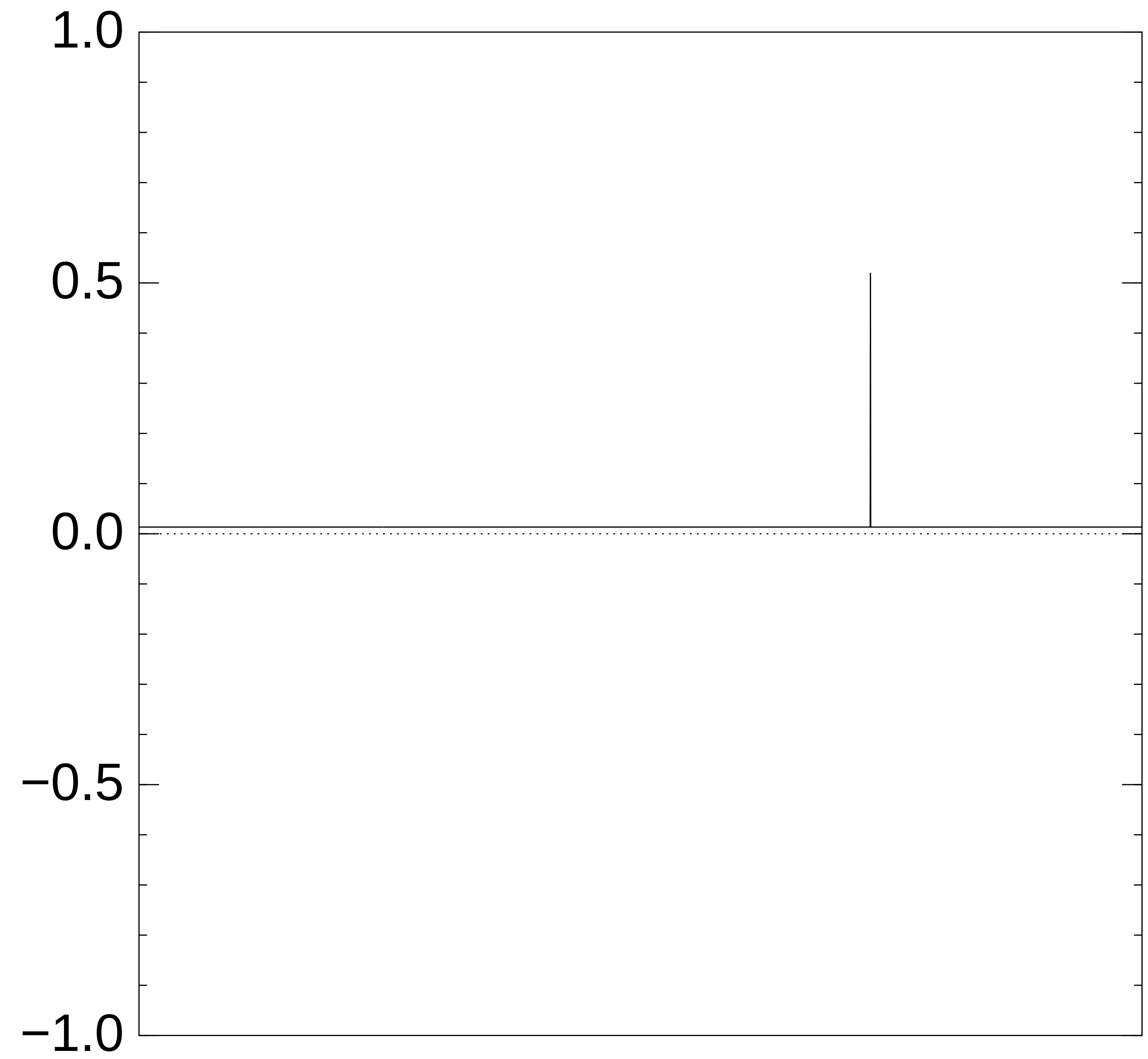
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

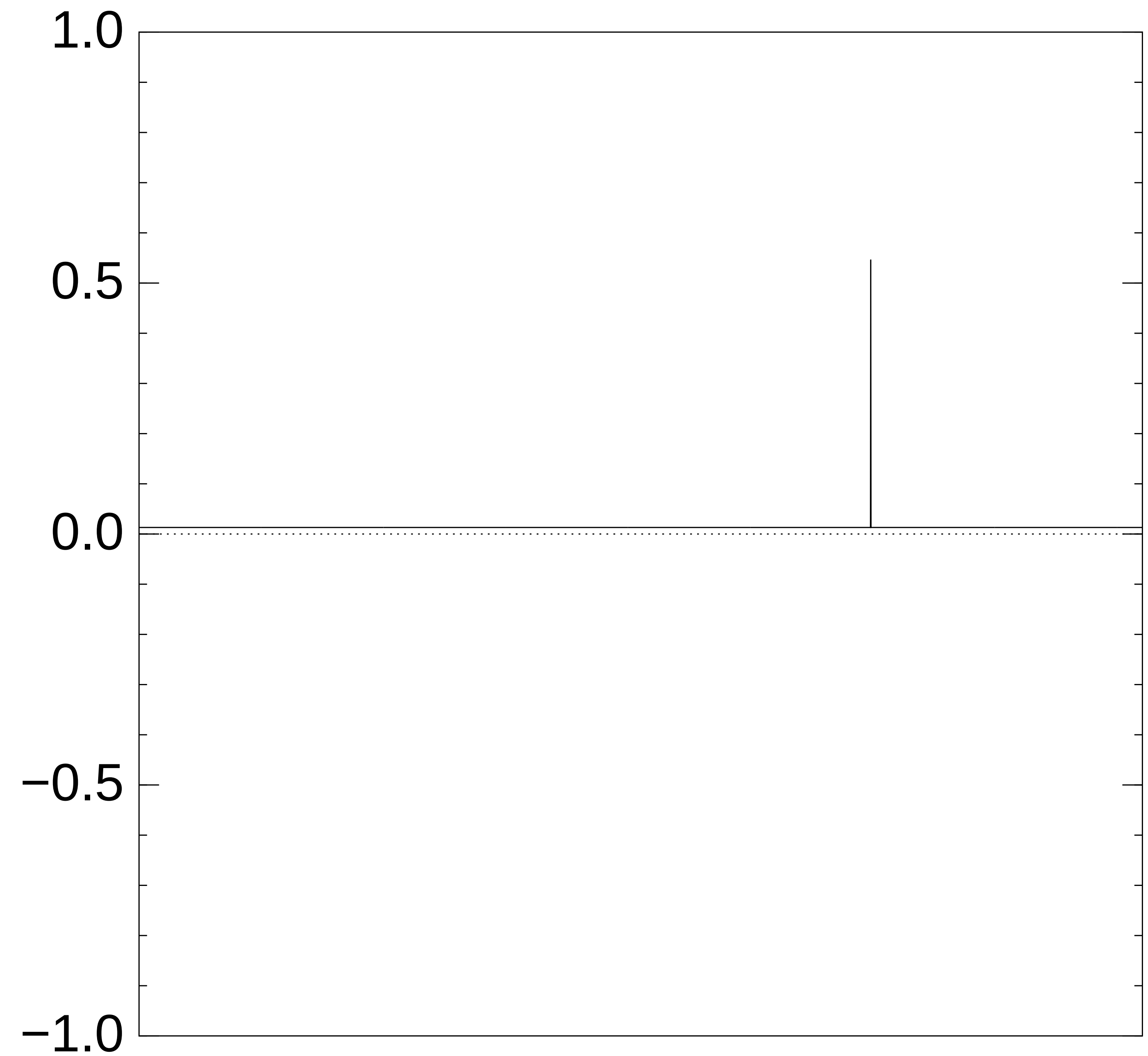
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

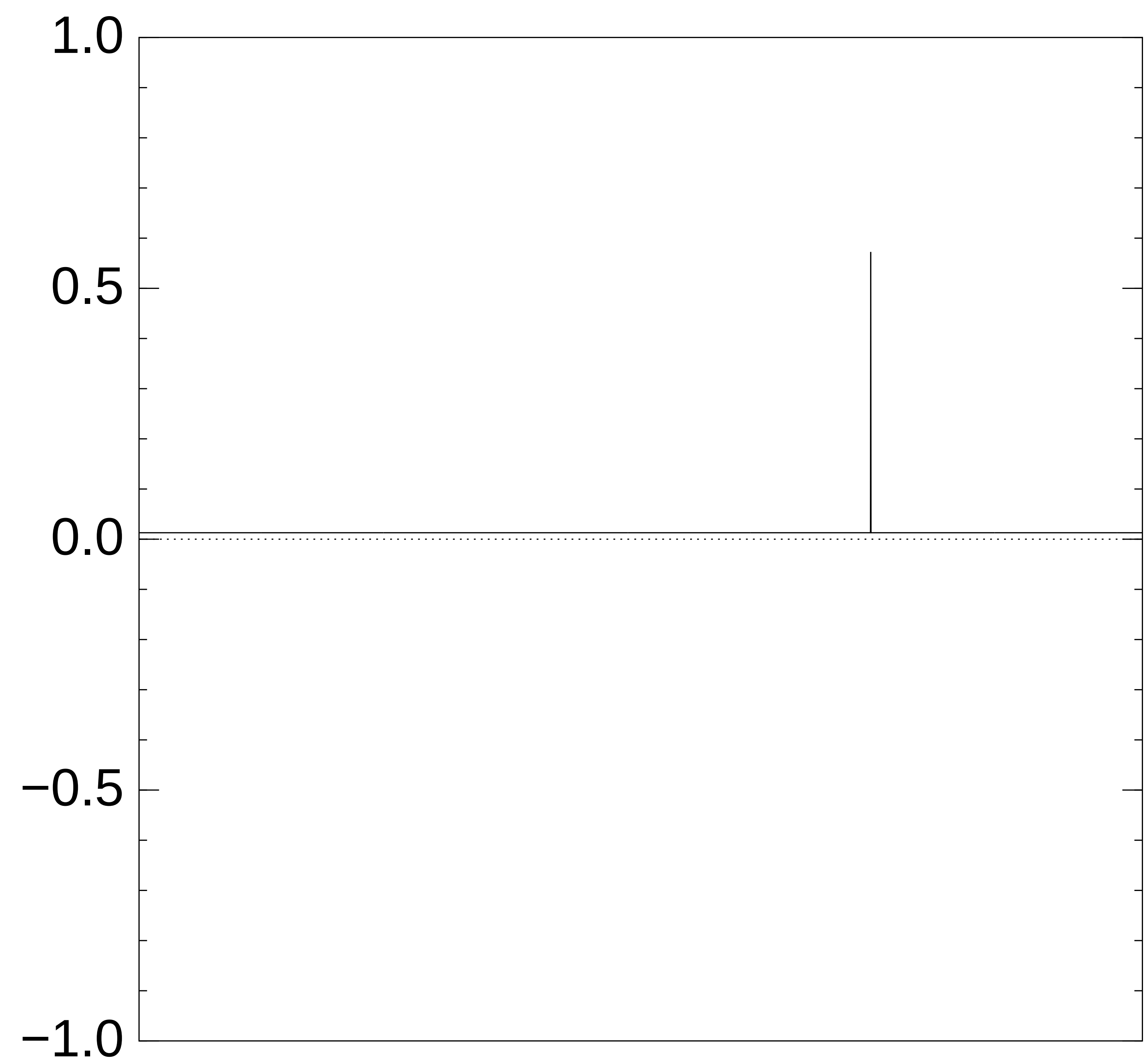
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $19 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

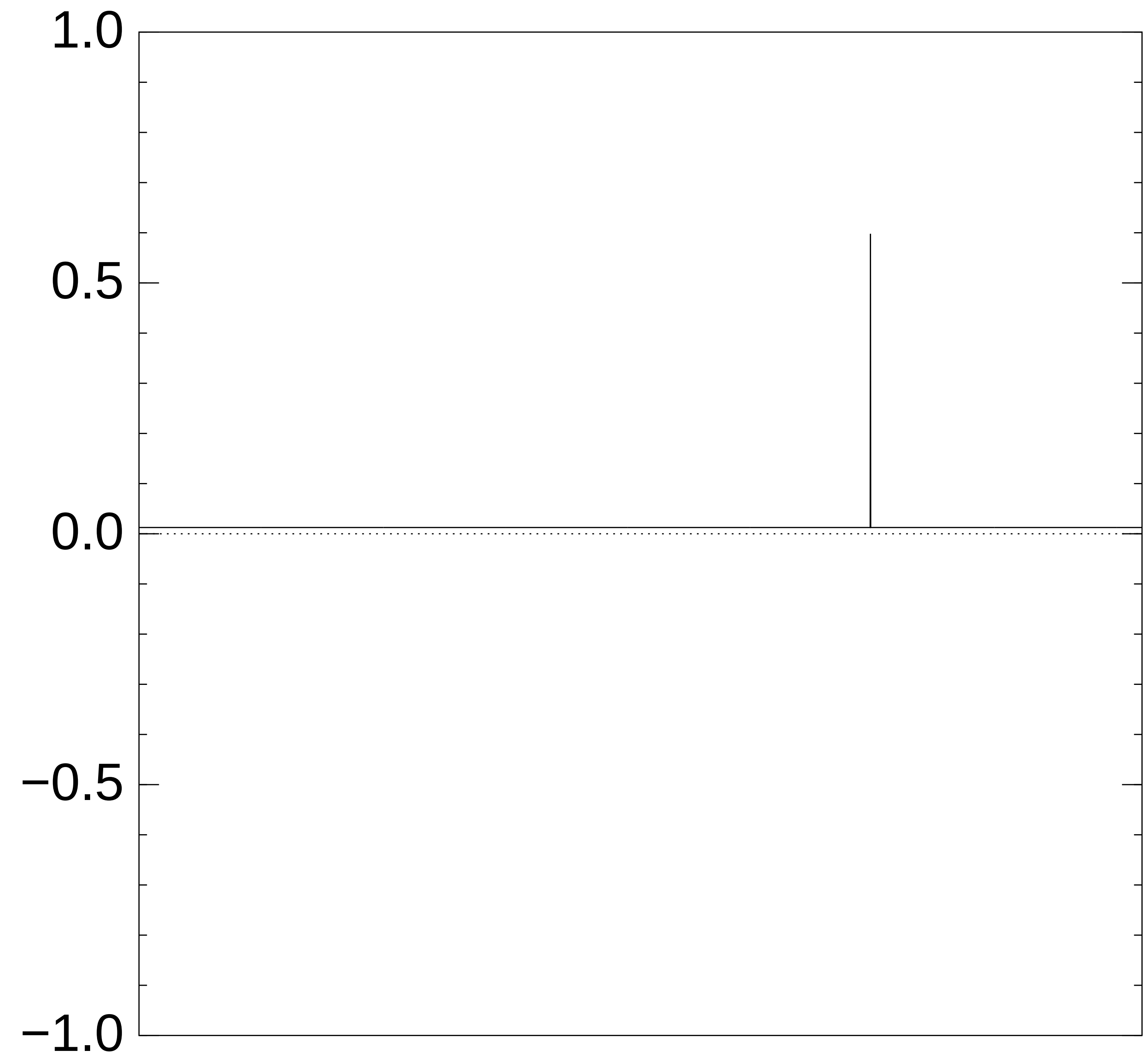
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $20 \times$ (Step $1 +$ Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

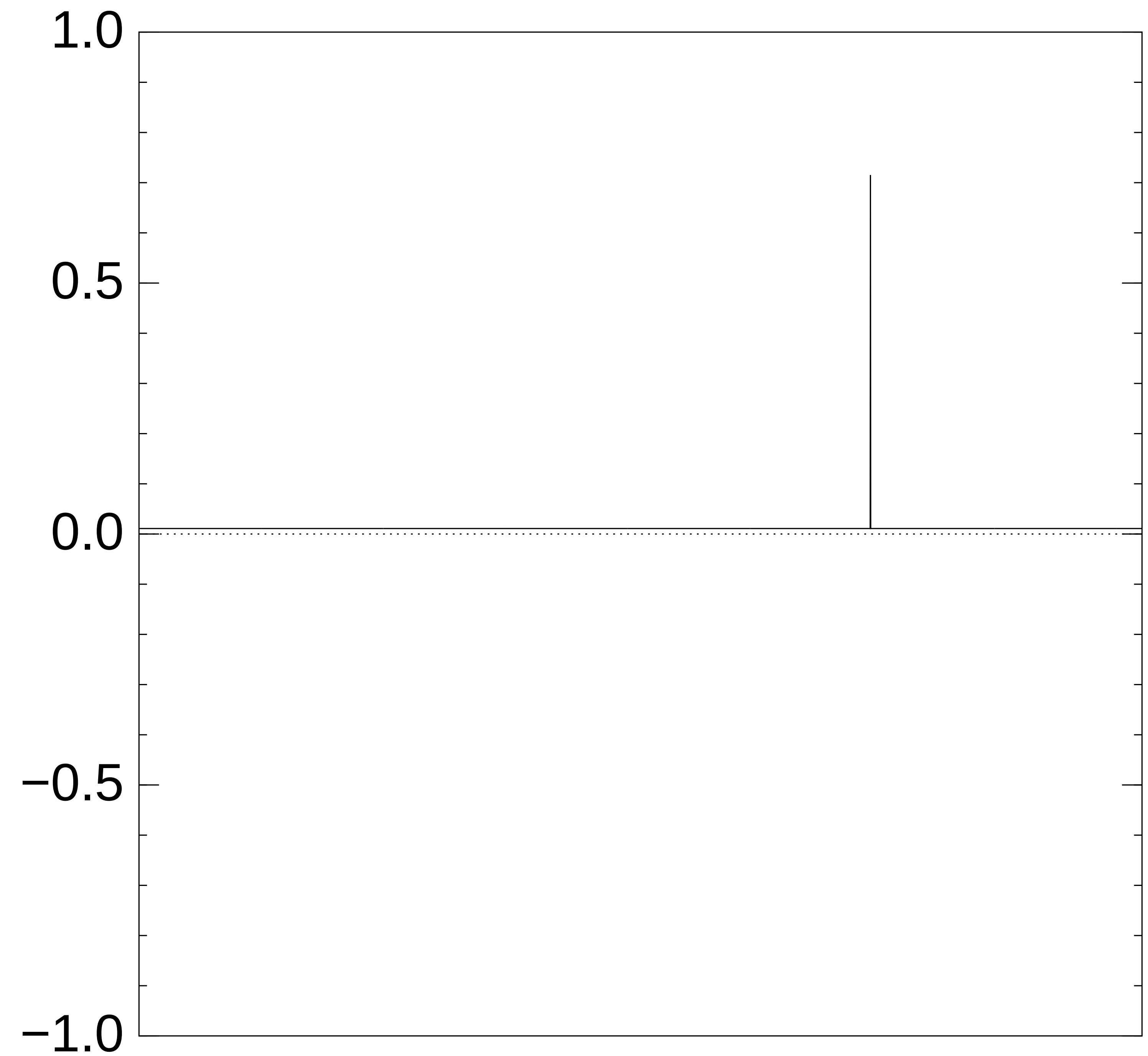
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $25 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

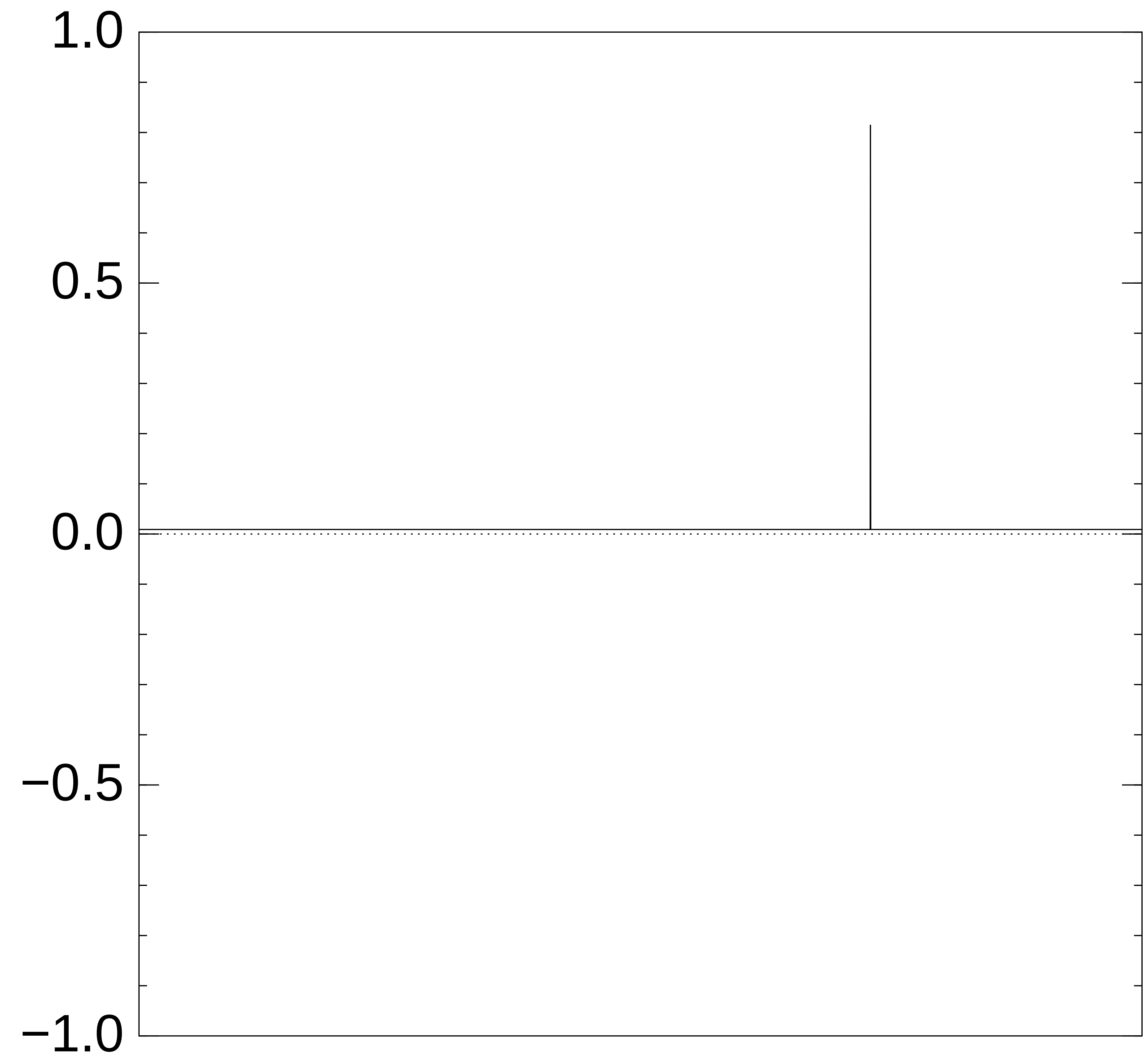
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $30 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
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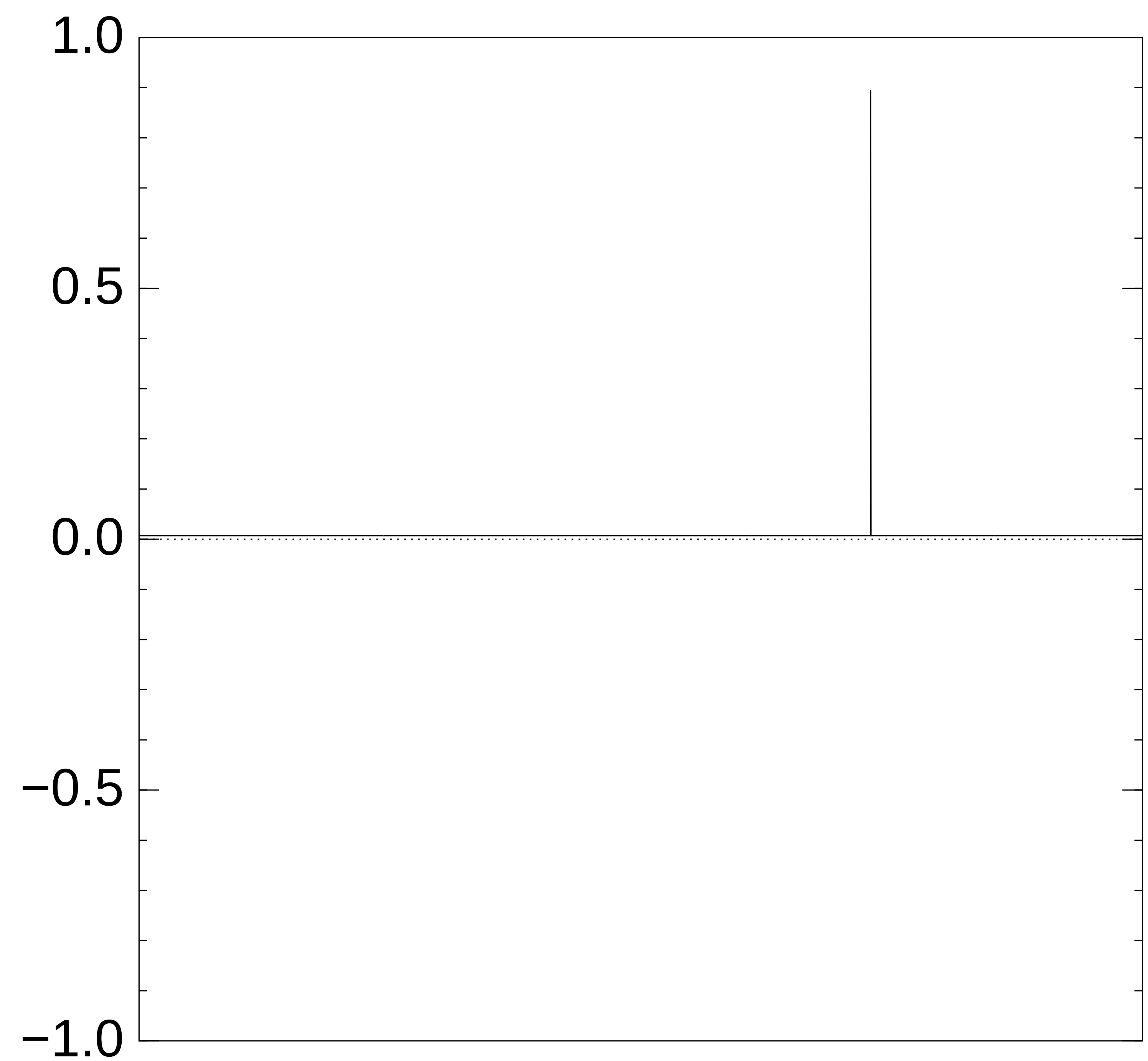
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $35 \times$ (Step 1 + Step 2):



Good moment to stop, measure.

Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

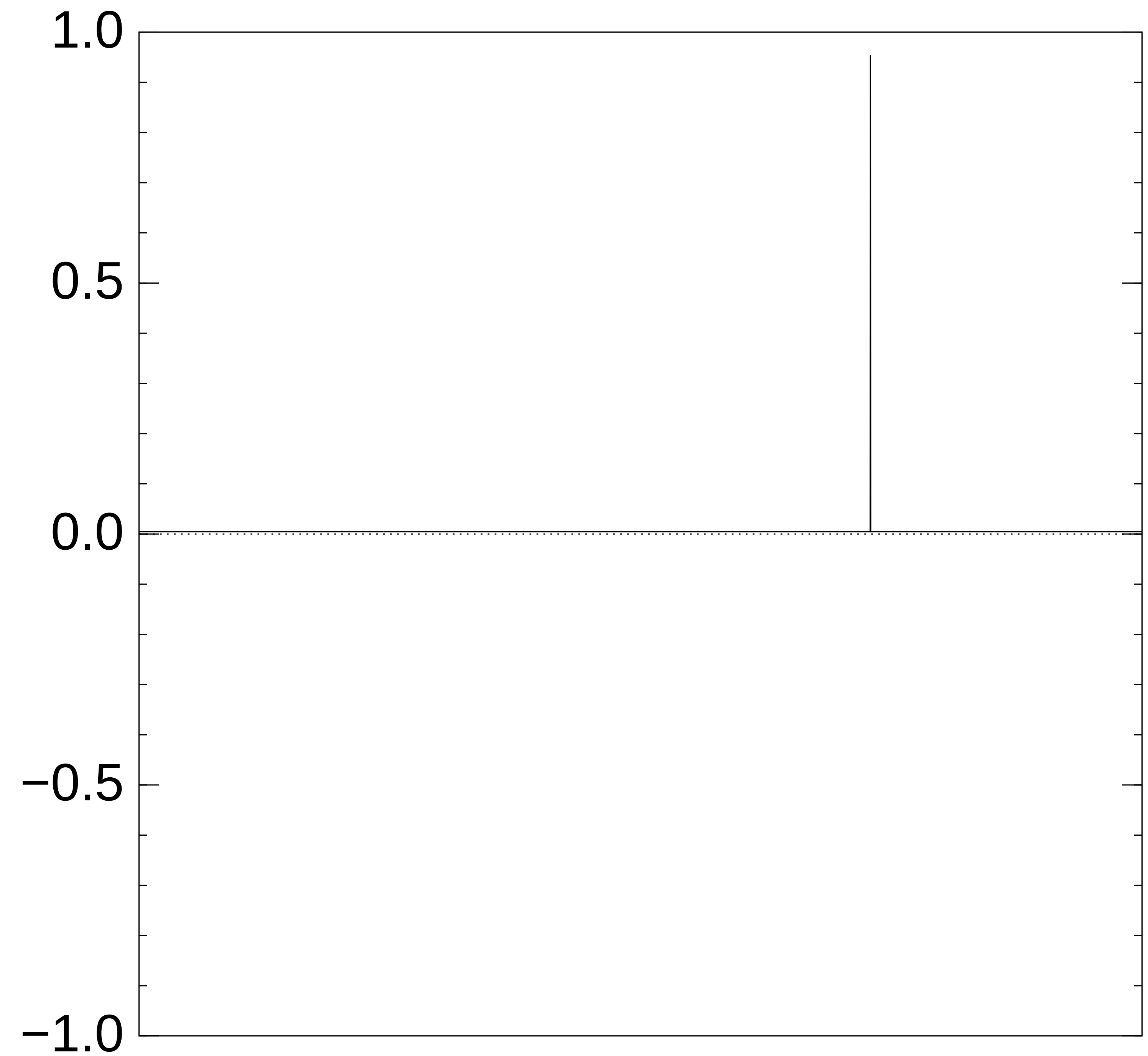
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $40 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

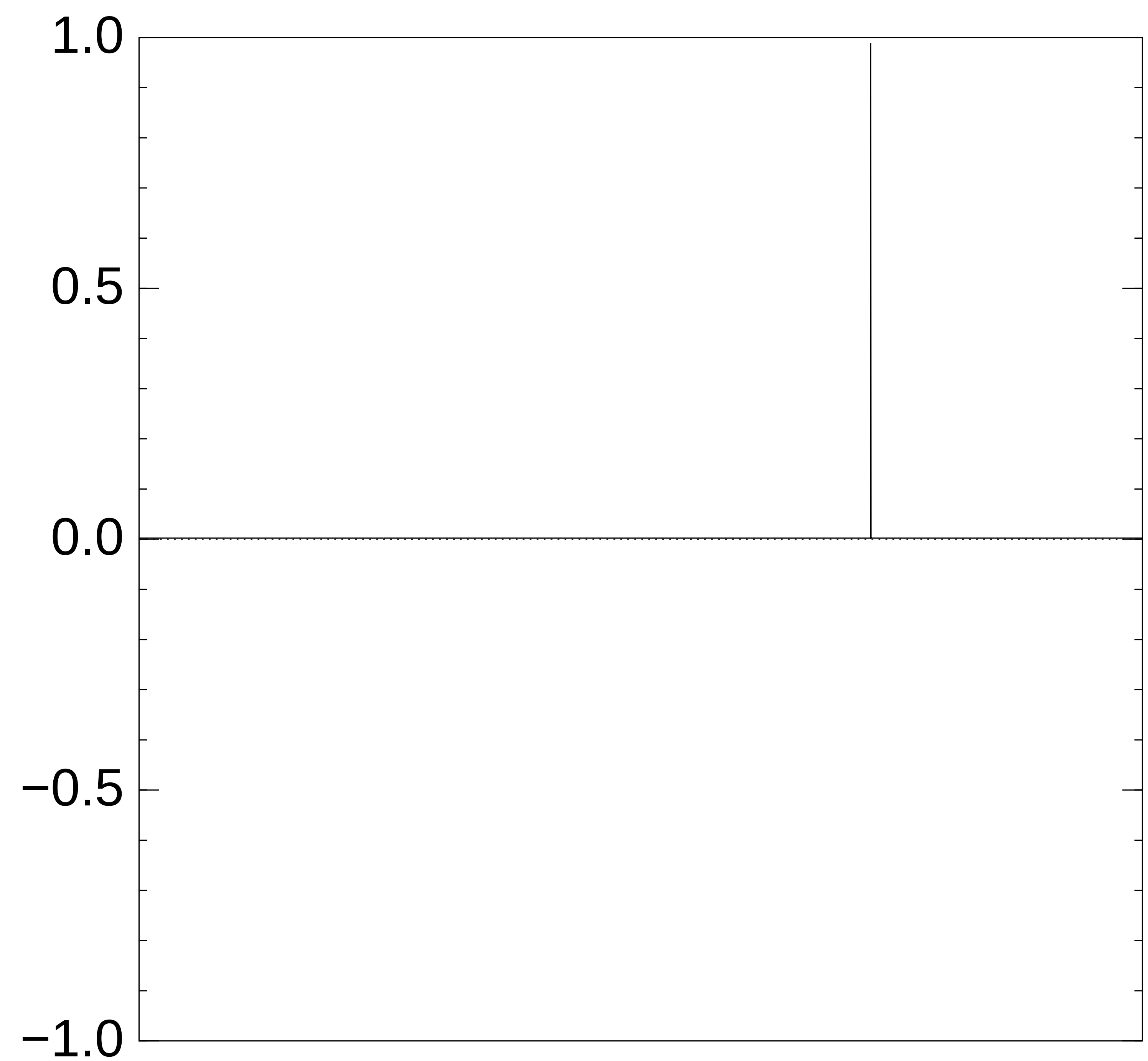
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $45 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

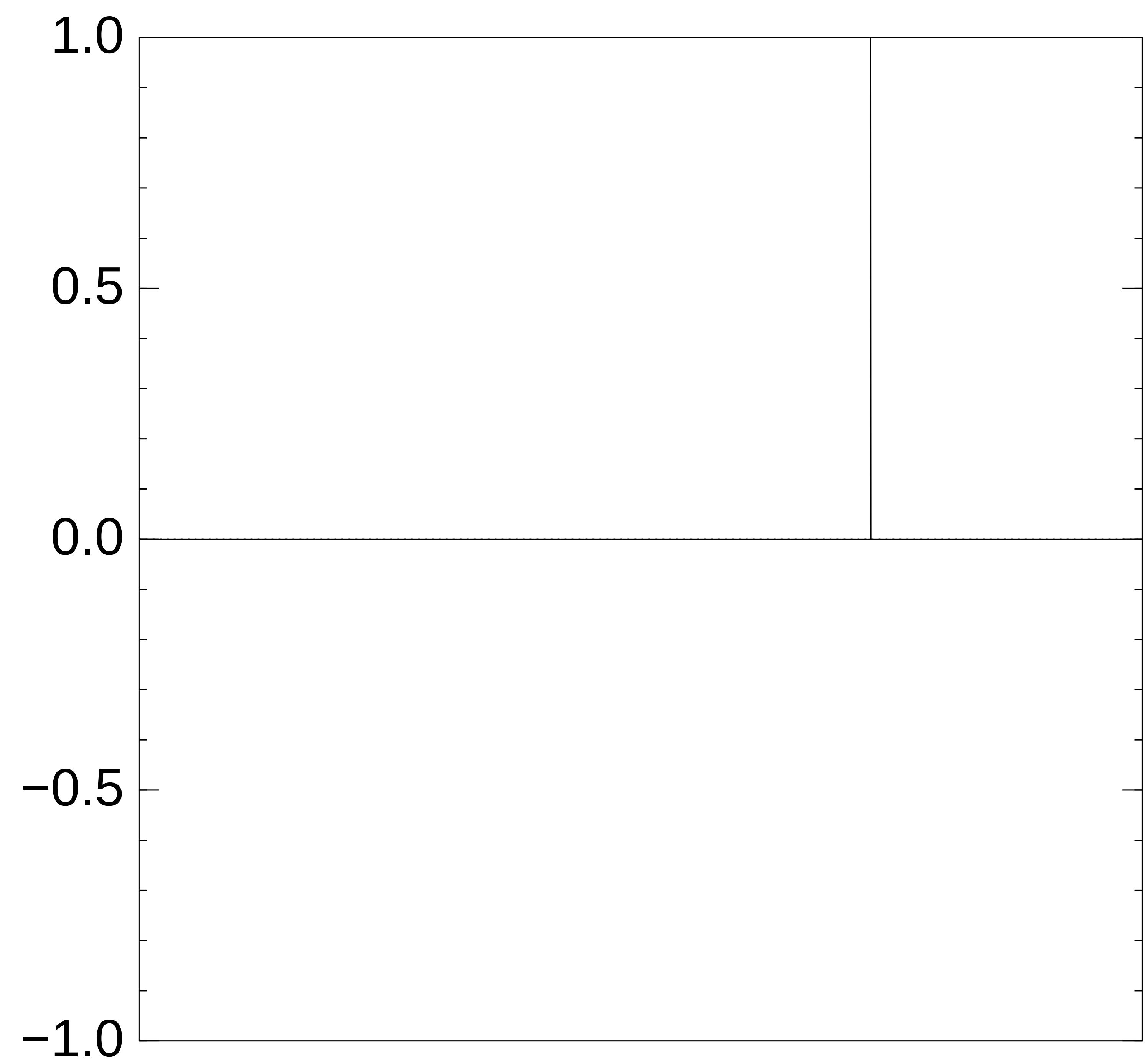
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $50 \times$ (Step 1 + Step 2):



Traditional stopping point.

Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
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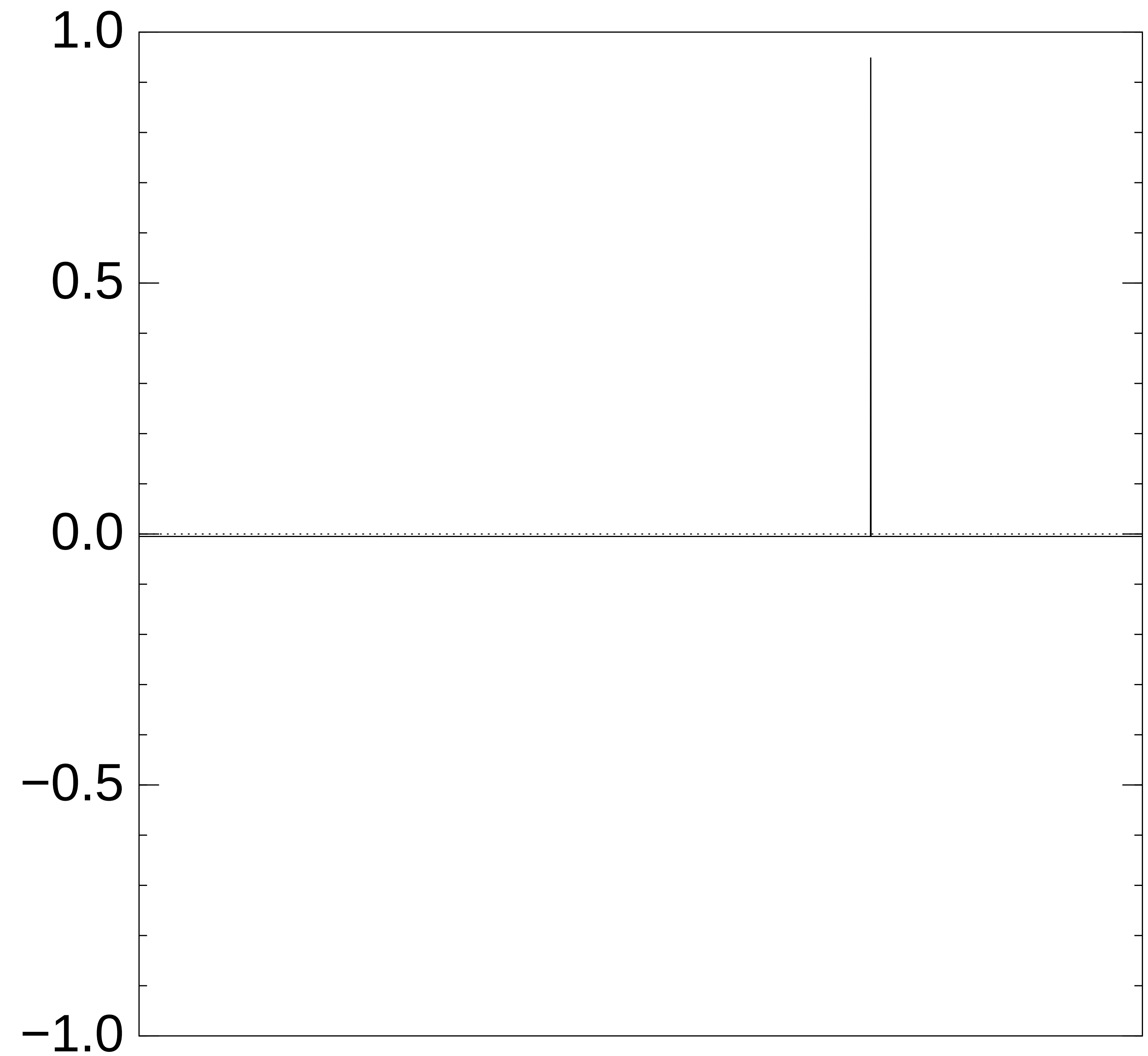
Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $60 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
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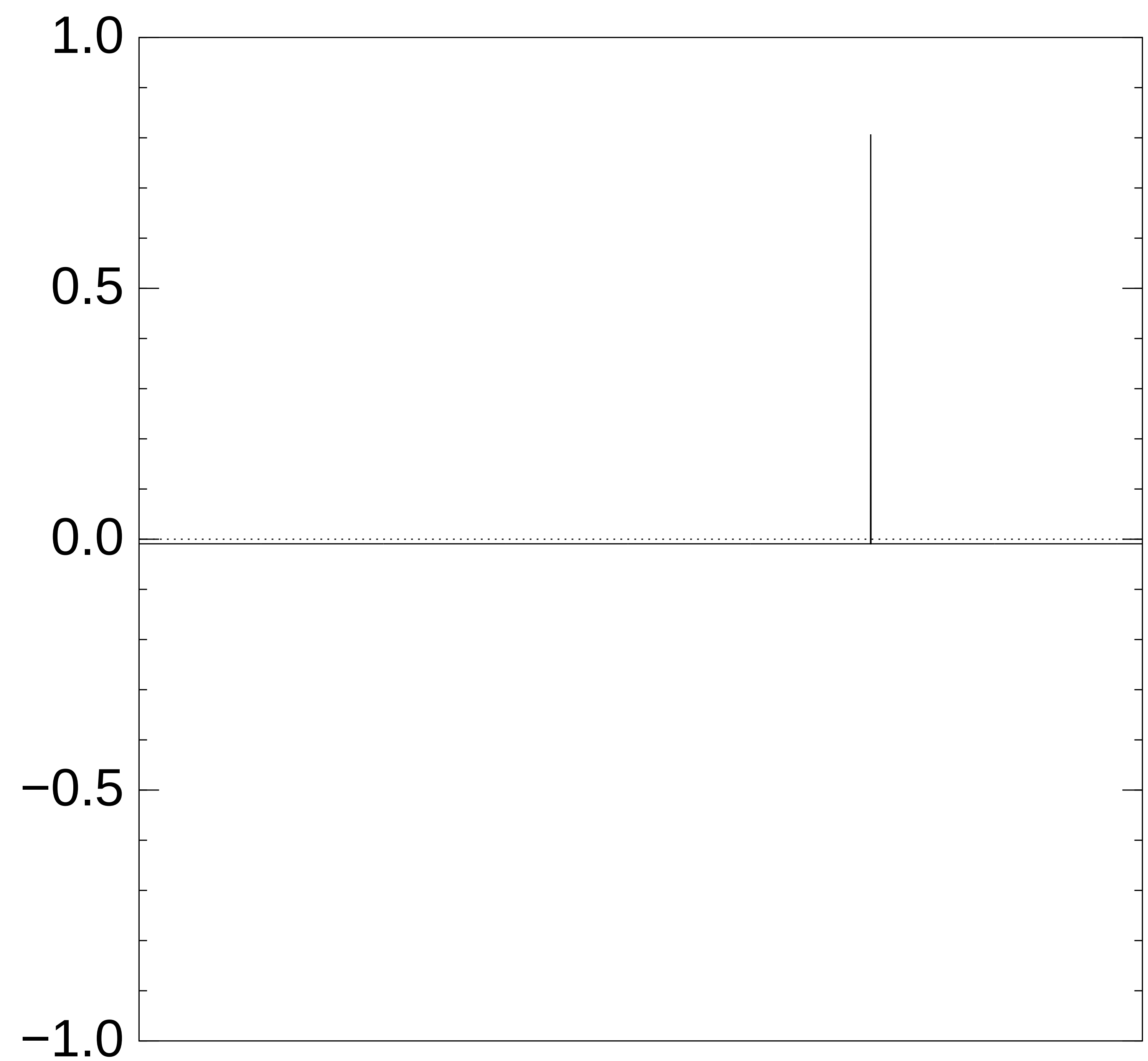
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $70 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

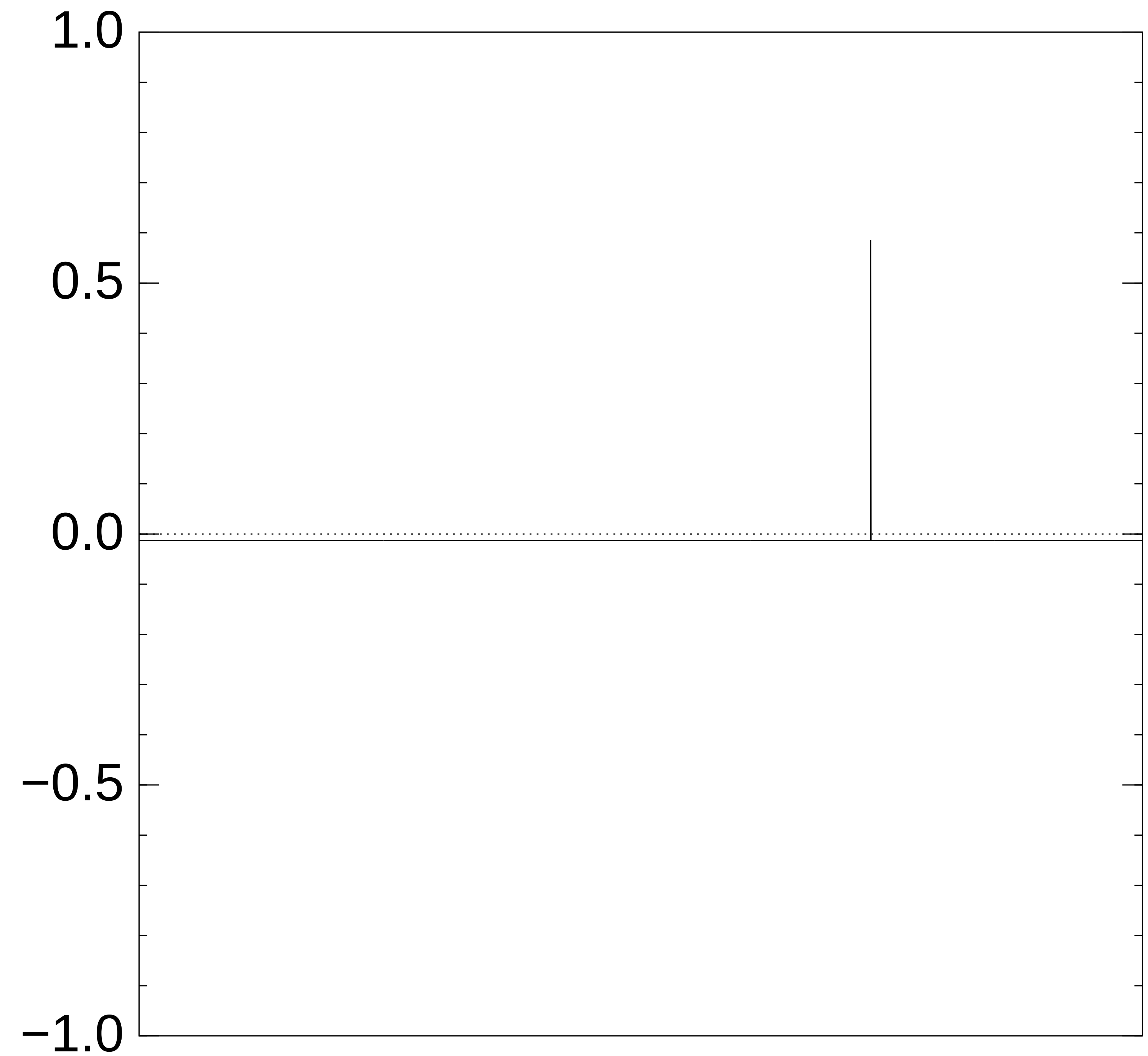
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $80 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

Step 1: Set $a \leftarrow b$ where
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Step 2: "Grover diffusion".
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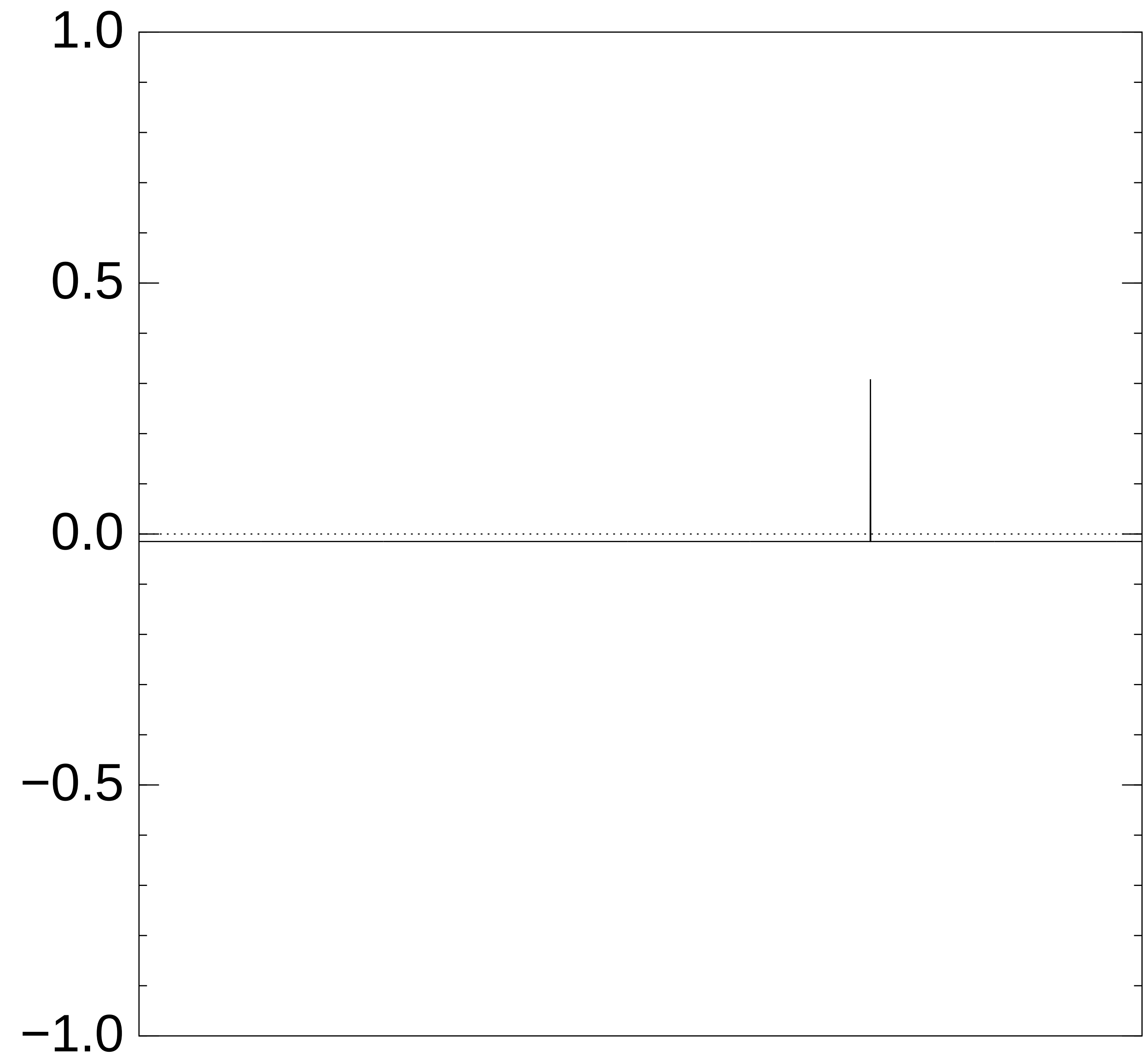
Repeat Step 1 + Step 2 about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition a over $q \in \{0,1\}^n$: $a_q = 2^{-n/2}$.

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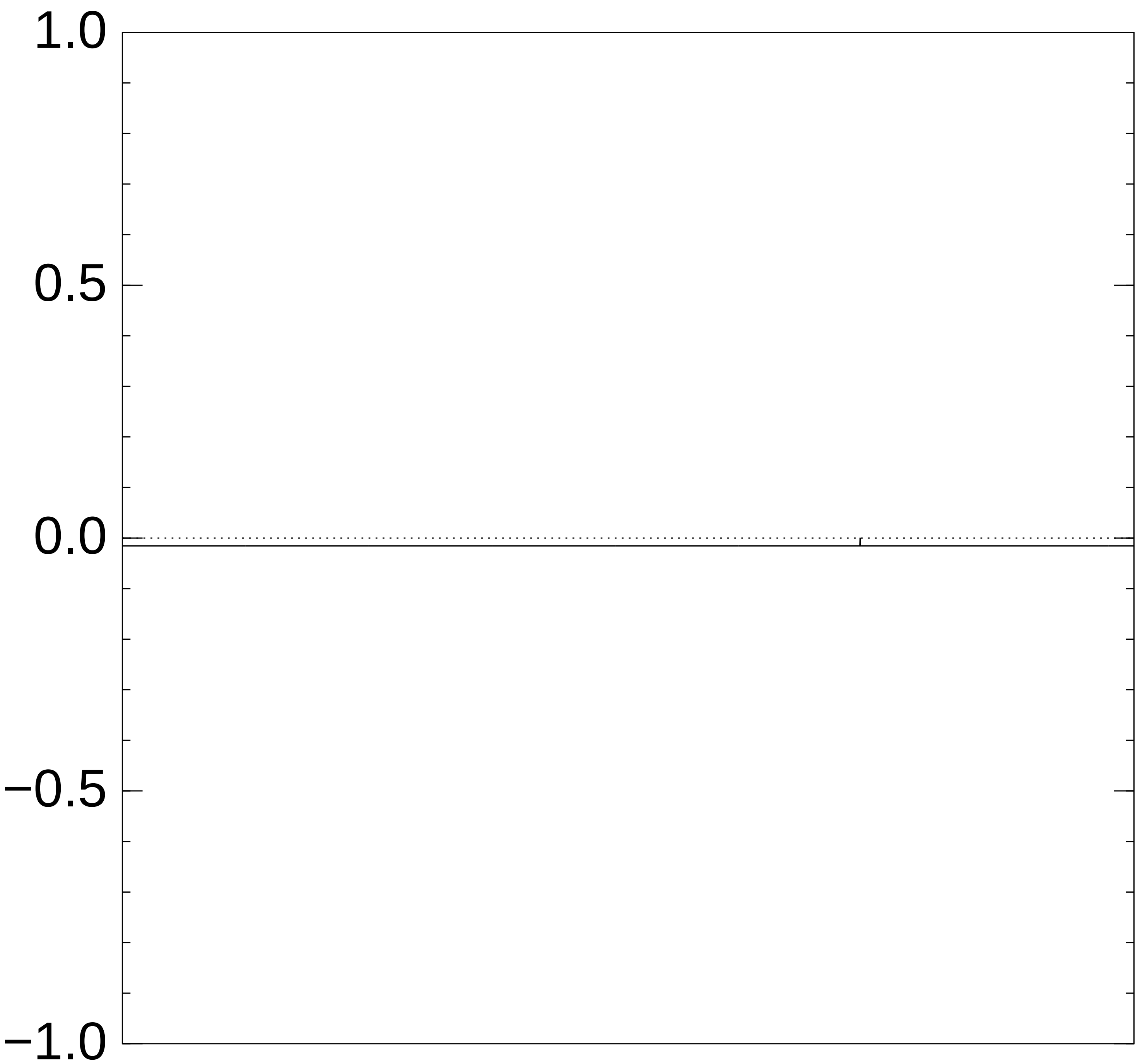
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{n/2}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

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Step $1 +$ Step 2
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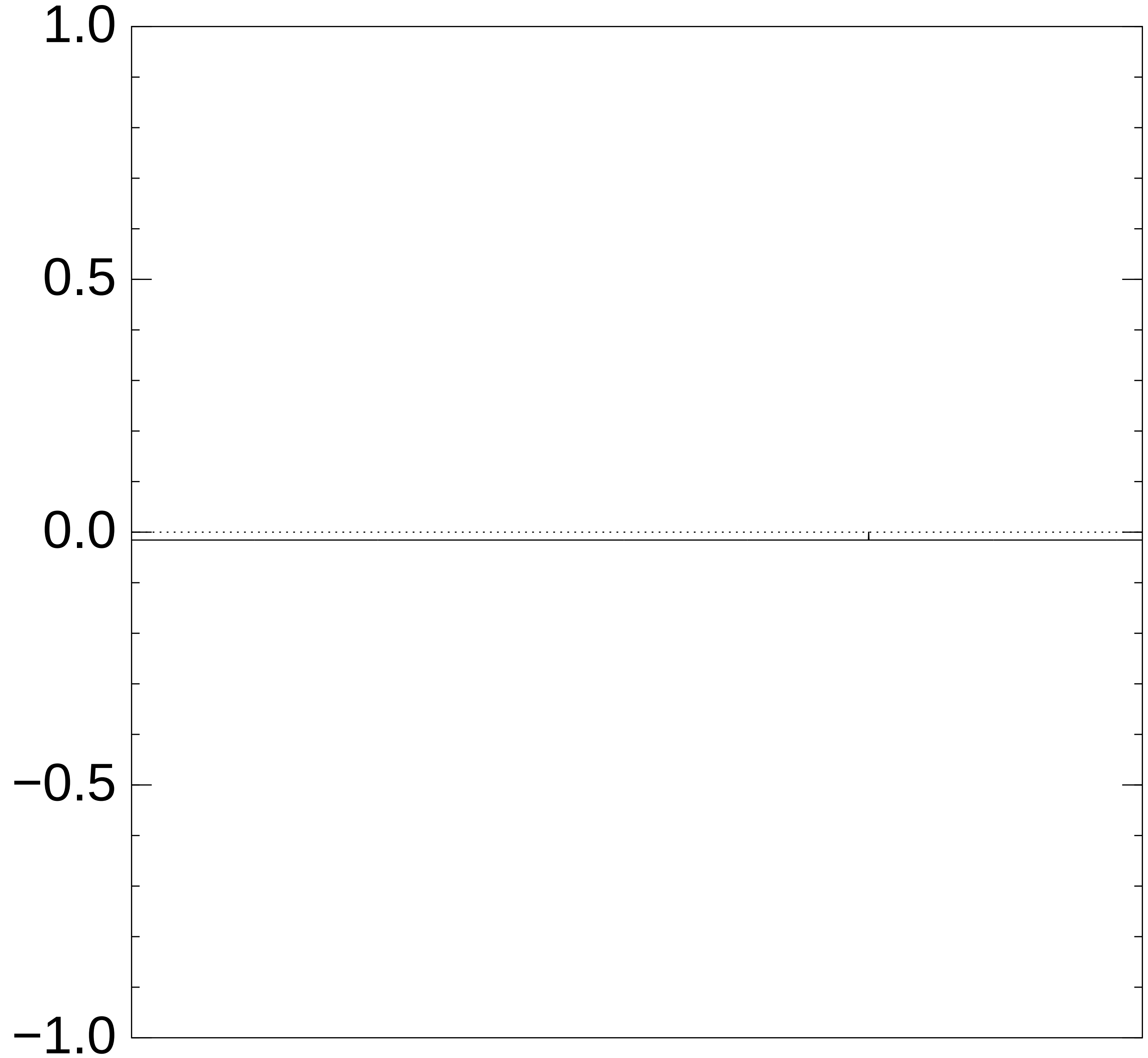
Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after $100 \times (\text{Step } 1 + \text{Step } 2)$:



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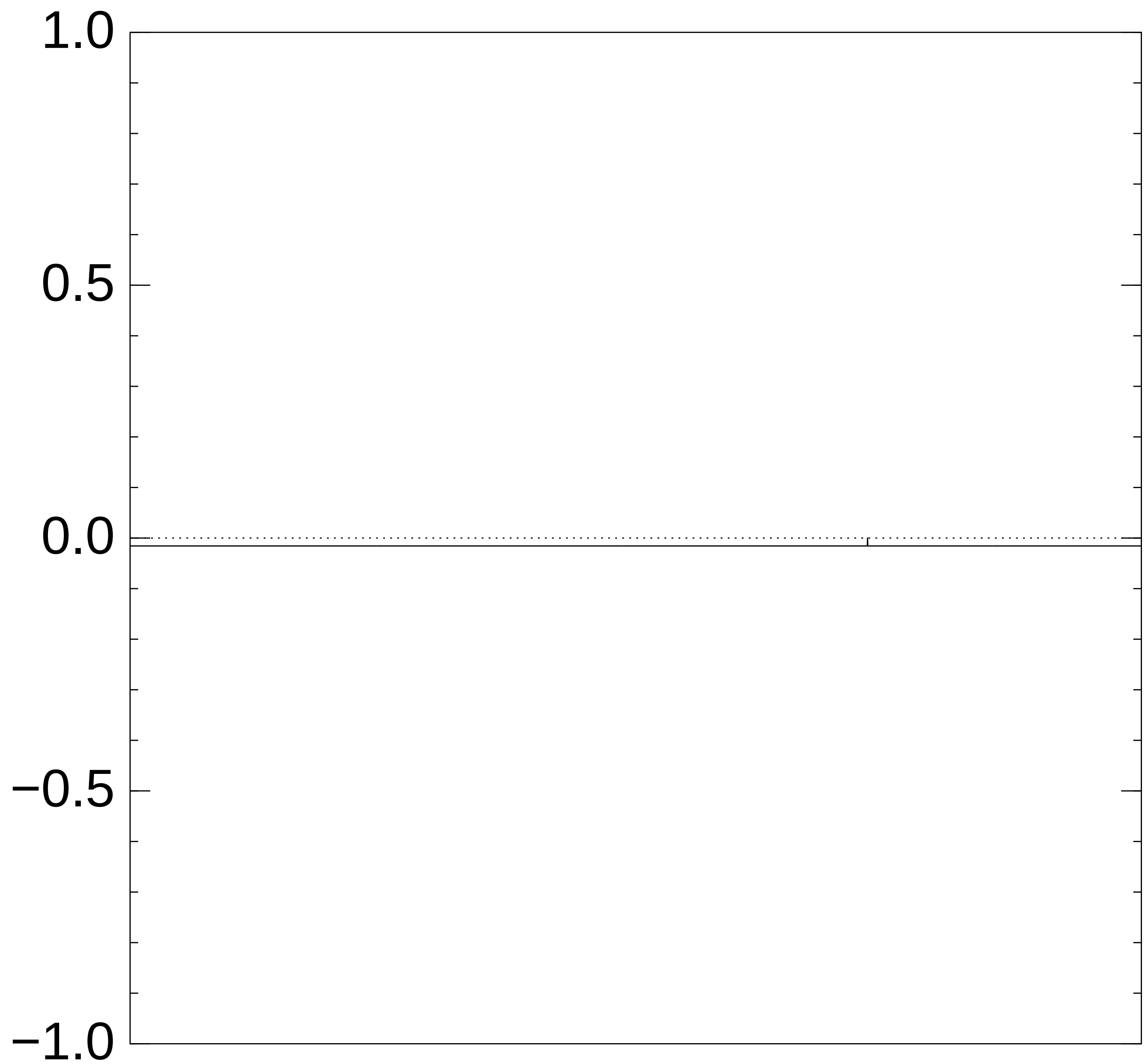

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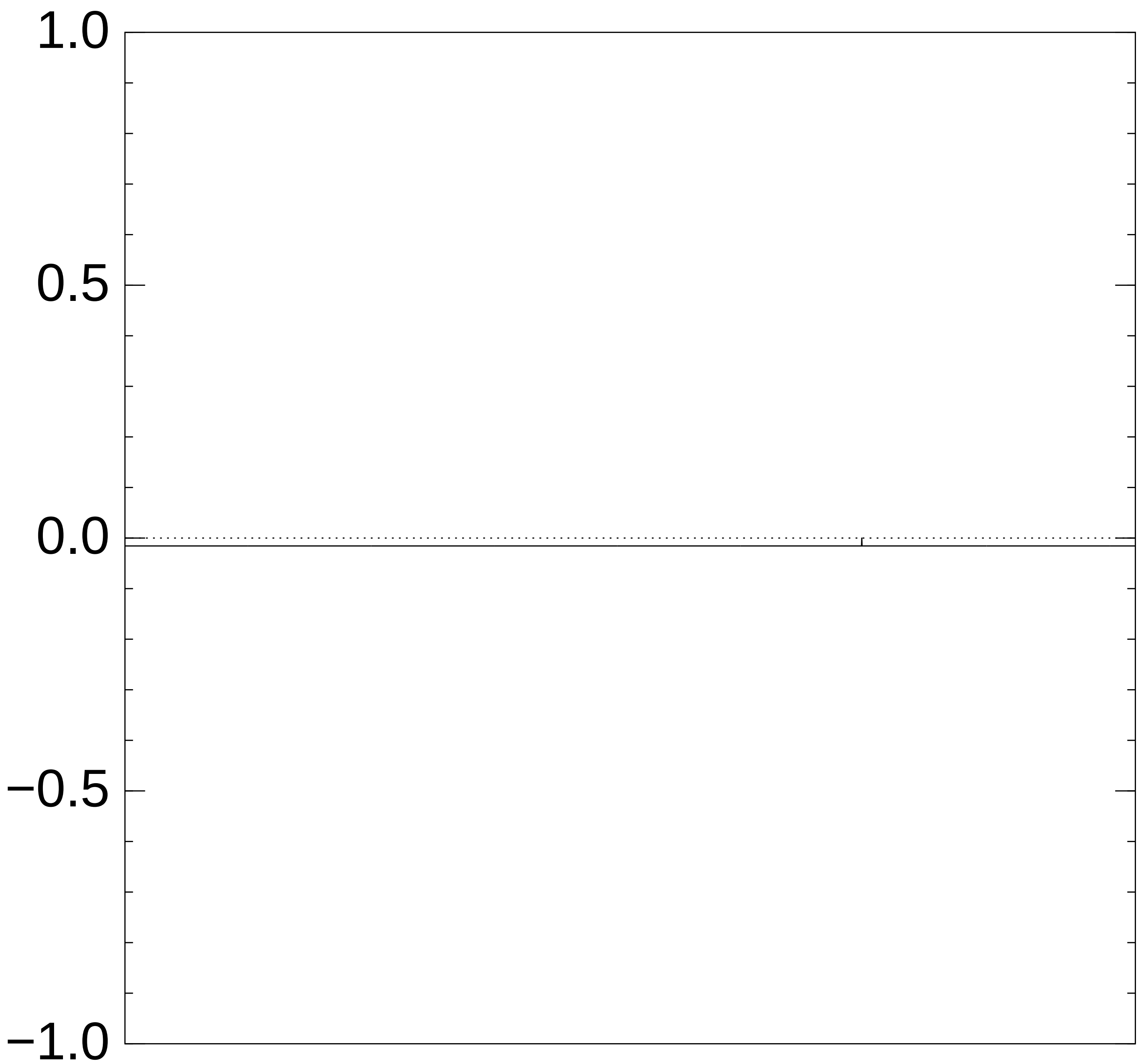

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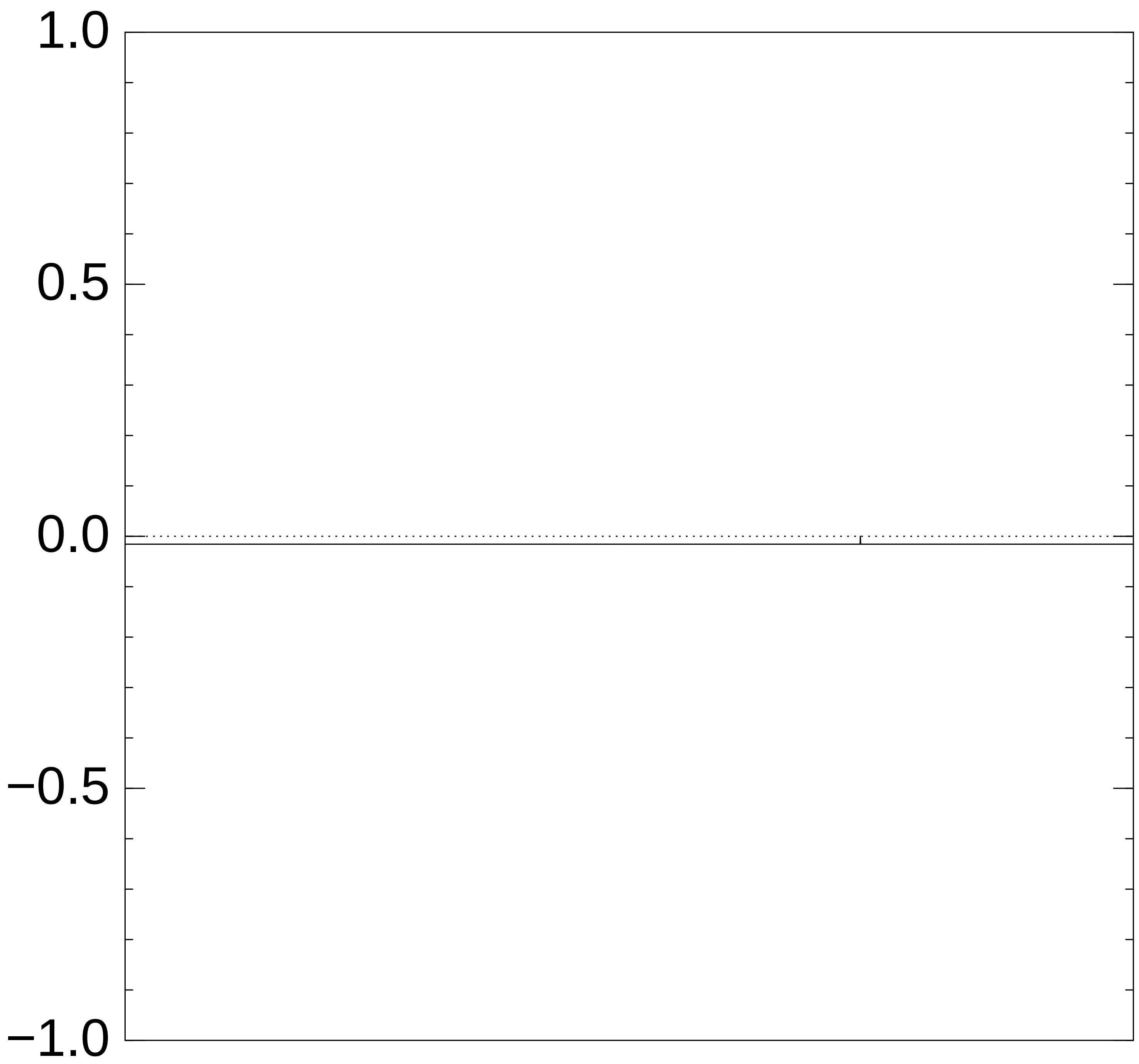

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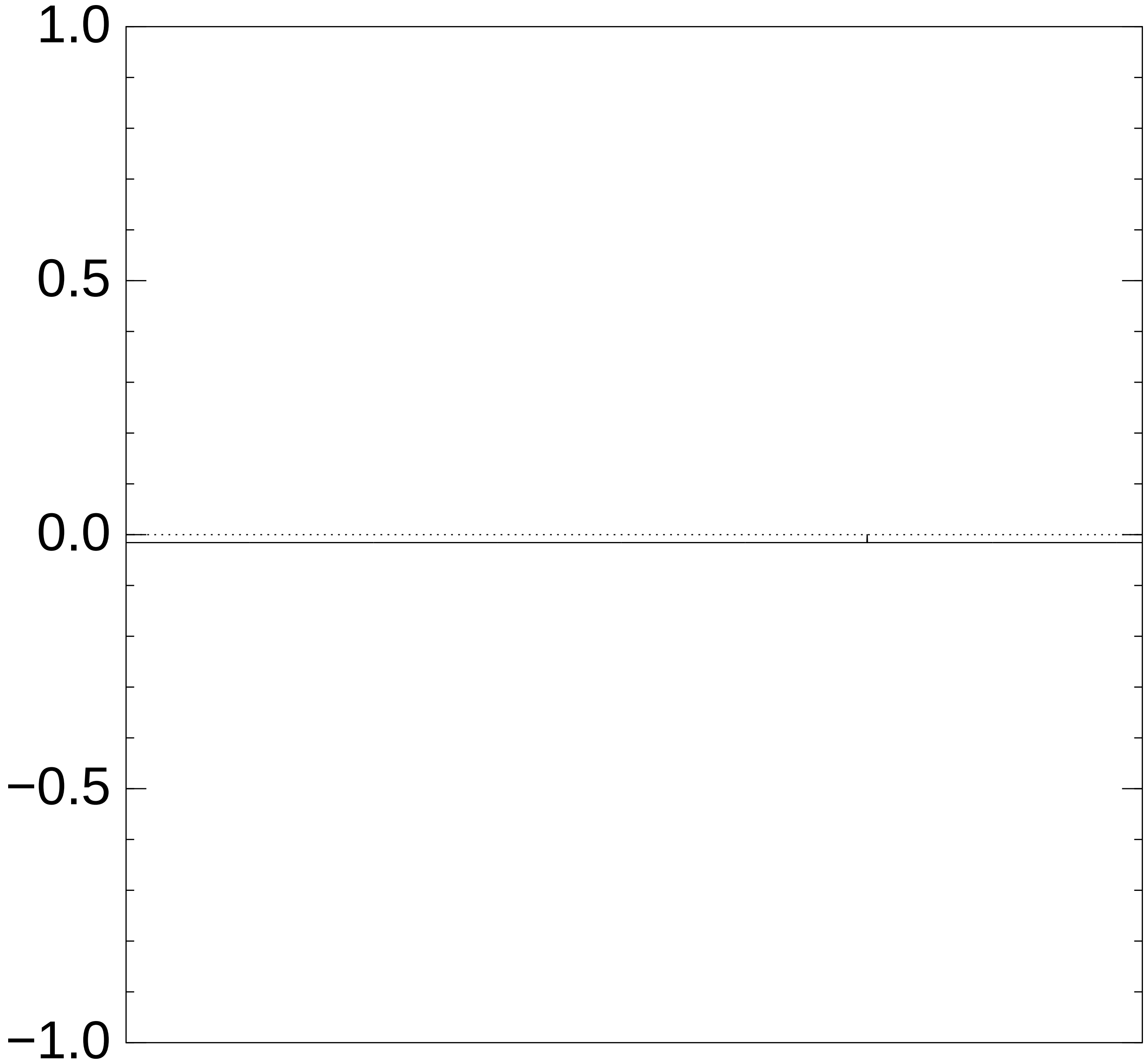

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Step 1 + Step 2
act linearly on this vector.

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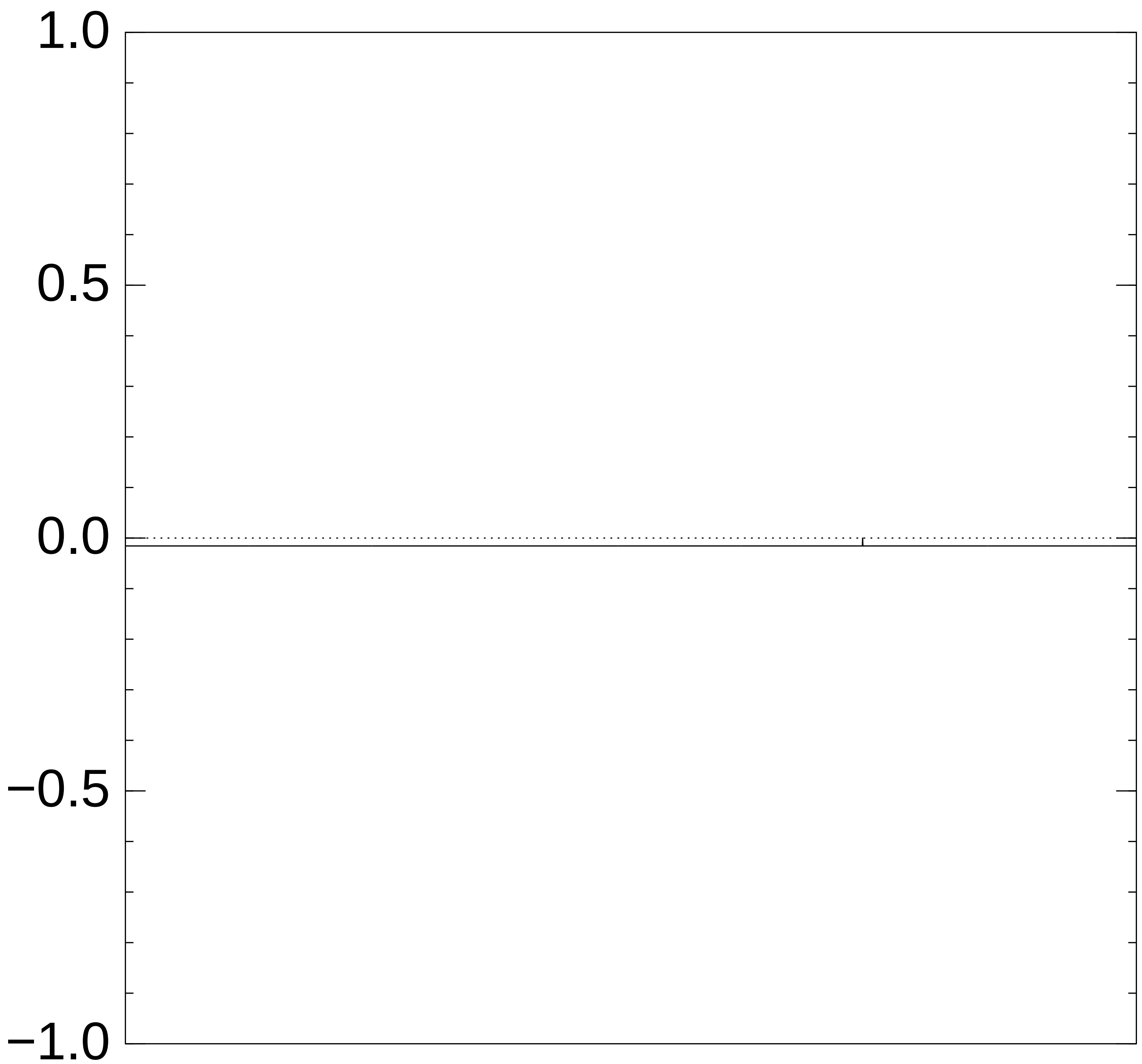

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Easily compute eigenvalues
and powers of this linear map
to understand evolution
of state of Grover's algorithm.
$\Rightarrow$ Probability is ≈ 1
after $\approx(\pi/4)2^{n/2}$ iterations.

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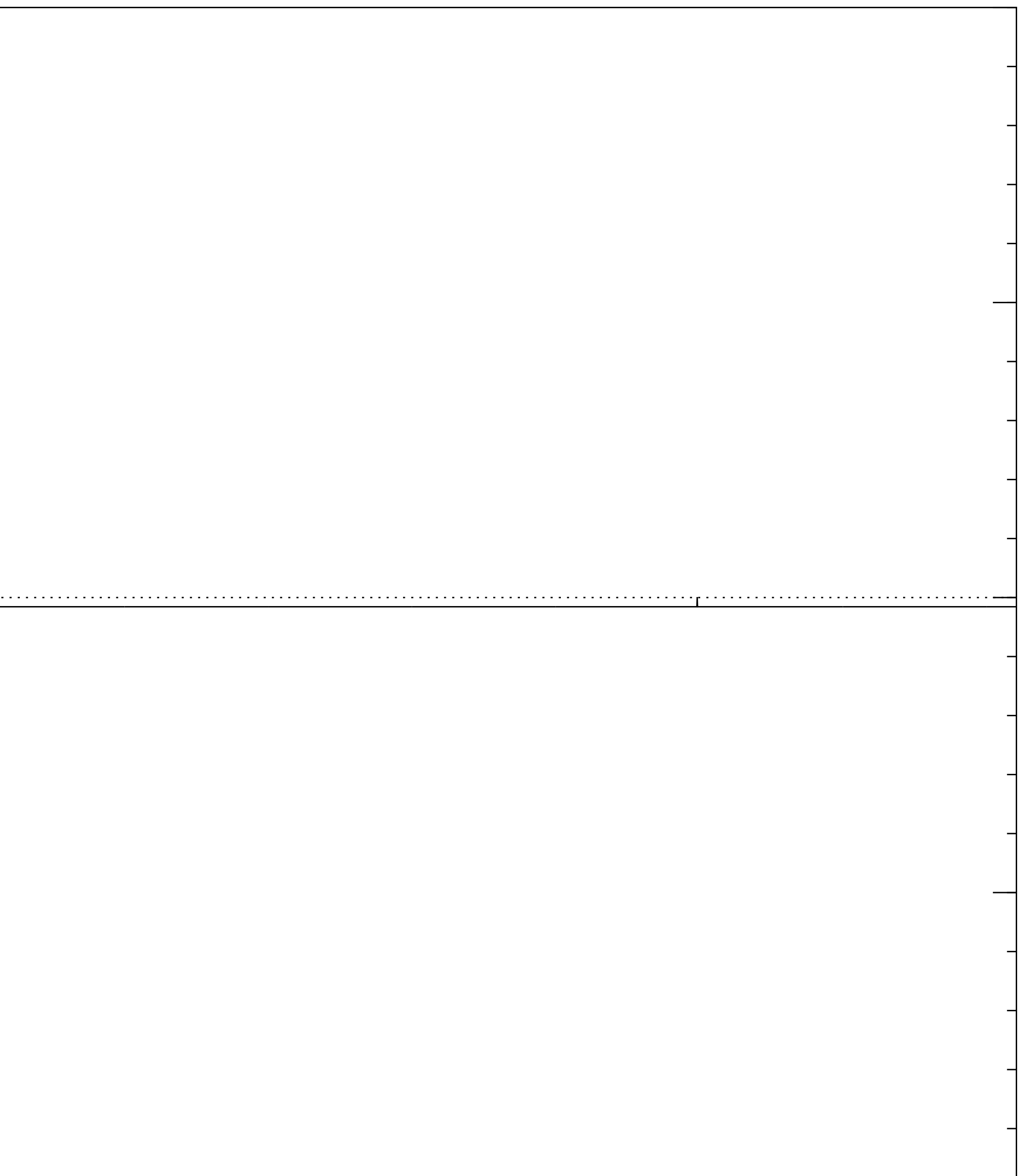

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Ambaini

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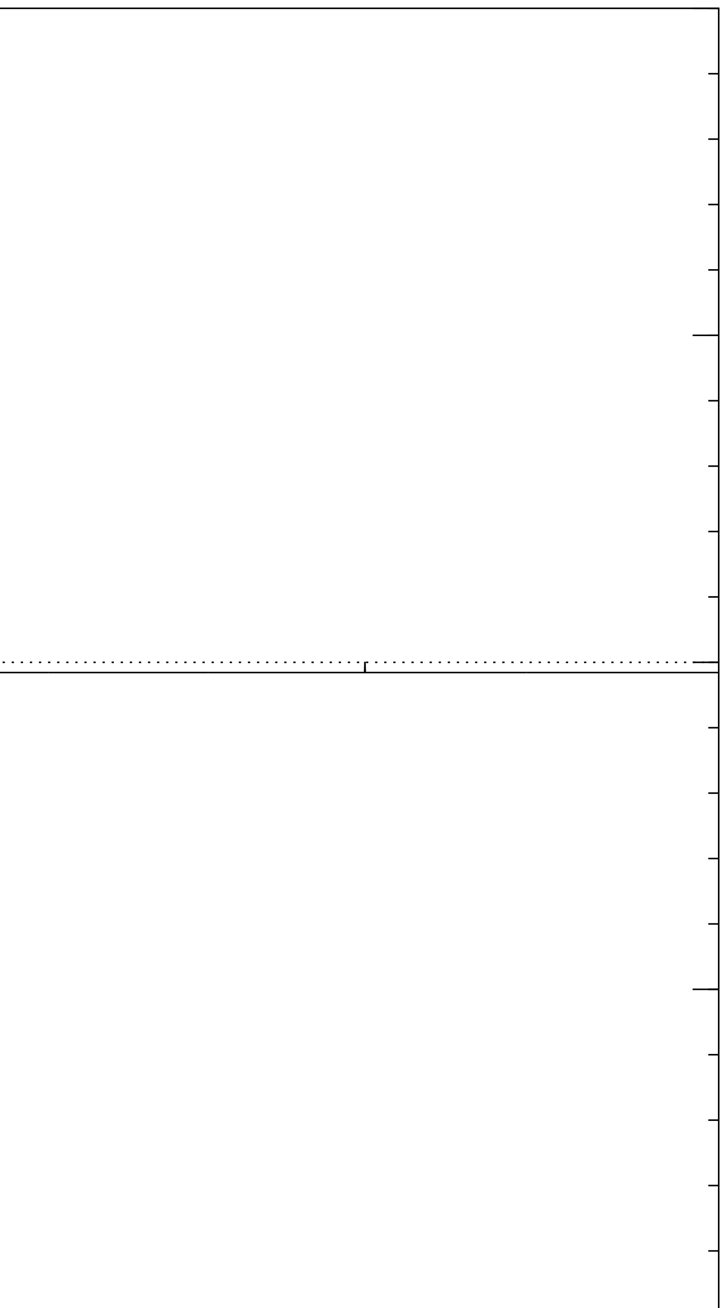

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<u>Ambainis's algorit</u>

Unique-collision-fi

Say f has n-bit in

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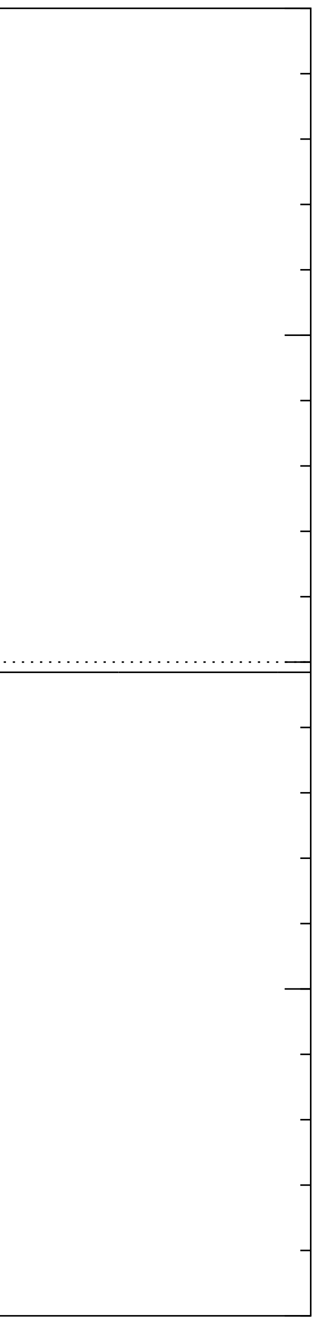

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Ambainis's algorithm

Unique-collision-finding prob
Say f has n-bit inputs,
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Problem: find this collision.

Cost 2^n: Define S as
the set of n-bit strings.
Compute $f(S)$, sort.

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Generalize to cost r, success probability $\approx(r/2^n)^2$:
Choose a set S of size r.
Compute $f(S)$, sort.

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Data structure $D(S)$ captur
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the set S; the multiset $f(S)$
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Ambainis's algorithm

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Very efficient to move from $D(S)$
to $D(T)$ if T is an **adjacent** set:
$\#S = \#T = r$, $\#(S \cap T) = r - 1$.

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2003 Ambainis, simplified 2007
Magniez–Nayak–Roland–Santha:
Create superposition of states
$(D(S), D(T))$ with adjacent S, T.
By a quantum walk
find S containing a collision.

s's algorithm

collision-finding problem:

as n-bit inputs,

one collision $\{p, q\}$:

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Define S as

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How the quantum

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if S contains
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For each S:
Diffuse $a_{S,}$

Now high probabil
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Cost $r + 2^n/\sqrt{r}$.

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Data structure $D(S)$ capturing
the generalized computation:
the set S; the multiset $f(S)$;
the number of collisions in S.

Very efficient to move from $D(S)$
to $D(T)$ if T is an **adjacent** set:
$\#S = \#T = r$, $\#(S \cap T) = r - 1$.

2003 Ambainis, simplified 2007
Magniez–Nayak–Roland–Santha:
Create superposition of states
$(D(S), D(T))$ with adjacent S, T.
By a quantum walk
find S containing a collision.

How the quantum walk work

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Classify (S, T) acc
$(\#(S \cap \{p, q\}), \#$
reduce a to low-di
Analyze evolution

e.g. $n = 15$, $r = 1$
0 negations and 0

Pr[class $(0, 0)$] $\approx$
Pr[class $(0, 1)$] $\approx$
Pr[class $(1, 0)$] $\approx$
Pr[class $(1, 1)$] $\approx$
Pr[class $(1, 2)$] $\approx$
Pr[class $(2, 1)$] $\approx$
Pr[class $(2, 2)$] $\approx$

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Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q$

reduce a to low-dim vector.

Analyze evolution of this vec

e.g. $n = 15$, $r = 1024$, after

0 negations and 0 diffusions

$\Pr[\text{class } (0, 0)] \approx 0.938; +$

$\Pr[\text{class } (0, 1)] \approx 0.000; +$

$\Pr[\text{class } (1, 0)] \approx 0.000; +$

$\Pr[\text{class } (1, 1)] \approx 0.060; +$

$\Pr[\text{class } (1, 2)] \approx 0.000; +$

$\Pr[\text{class } (2, 1)] \approx 0.000; +$

$\Pr[\text{class } (2, 2)] \approx 0.001; +$

Right column is sign of $a_{S,T}$

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Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

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Now high probability

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Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

1 negation and 46 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.935; +$

$\Pr[\text{class } (0, 1)] \approx 0.000; +$

$\Pr[\text{class } (1, 0)] \approx 0.000; -$

$\Pr[\text{class } (1, 1)] \approx 0.057; +$

$\Pr[\text{class } (1, 2)] \approx 0.000; +$

$\Pr[\text{class } (2, 1)] \approx 0.000; -$

$\Pr[\text{class } (2, 2)] \approx 0.008; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
Negate $a_{S,T}$
if S contains collision.
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Now high probability
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Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
2 negations and 92 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.918; +$
$\Pr[\text{class } (0, 1)] \approx 0.001; +$
$\Pr[\text{class } (1, 0)] \approx 0.000; -$
$\Pr[\text{class } (1, 1)] \approx 0.059; +$
$\Pr[\text{class } (1, 2)] \approx 0.001; +$
$\Pr[\text{class } (2, 1)] \approx 0.000; -$
$\Pr[\text{class } (2, 2)] \approx 0.022; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
  Negate $a_{S,T}$
    if S contains collision.
  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
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      Diffuse $a_{S,T}$ across all S.
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Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
3 negations and 138 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.897; +$
$\Pr[\text{class } (0, 1)] \approx 0.001; +$
$\Pr[\text{class } (1, 0)] \approx 0.000; -$
$\Pr[\text{class } (1, 1)] \approx 0.058; +$
$\Pr[\text{class } (1, 2)] \approx 0.002; +$
$\Pr[\text{class } (2, 1)] \approx 0.000; +$
$\Pr[\text{class } (2, 2)] \approx 0.042; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

  Negate $a_{S,T}$

    if S contains collision.

  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

    For each T:

      Diffuse $a_{S,T}$ across all S.

    For each S:

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Now high probability that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to $(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$; reduce a to low-dim vector. Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after 4 negations and 184 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.873; +$

$\Pr[\text{class } (0,1)] \approx 0.001; +$

$\Pr[\text{class } (1,0)] \approx 0.000; -$

$\Pr[\text{class } (1,1)] \approx 0.054; +$

$\Pr[\text{class } (1,2)] \approx 0.002; +$

$\Pr[\text{class } (2,1)] \approx 0.000; +$

$\Pr[\text{class } (2,2)] \approx 0.070; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

  Negate $a_{S,T}$

    if S contains collision.

  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

    For each T:

      Diffuse $a_{S,T}$ across all S.

    For each S:

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Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

5 negations and 230 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.838; +$

$\Pr[\text{class } (0, 1)] \approx 0.001; +$

$\Pr[\text{class } (1, 0)] \approx 0.001; -$

$\Pr[\text{class } (1, 1)] \approx 0.054; +$

$\Pr[\text{class } (1, 2)] \approx 0.003; +$

$\Pr[\text{class } (2, 1)] \approx 0.000; +$

$\Pr[\text{class } (2, 2)] \approx 0.104; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

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Now high probability

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Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

6 negations and 276 diffusions:

$\Pr[\text{class }(0, 0)] \approx 0.800; +$

$\Pr[\text{class }(0, 1)] \approx 0.001; +$

$\Pr[\text{class }(1, 0)] \approx 0.001; -$

$\Pr[\text{class }(1, 1)] \approx 0.051; +$

$\Pr[\text{class }(1, 2)] \approx 0.006; +$

$\Pr[\text{class }(2, 1)] \approx 0.000; +$

$\Pr[\text{class }(2, 2)] \approx 0.141; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
  Negate $a_{S,T}$
    if S contains collision.
  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
    For each T:
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Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
7 negations and 322 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.758; +$
$\Pr[\text{class } (0, 1)] \approx 0.002; +$
$\Pr[\text{class } (1, 0)] \approx 0.001; -$
$\Pr[\text{class } (1, 1)] \approx 0.047; +$
$\Pr[\text{class } (1, 2)] \approx 0.007; +$
$\Pr[\text{class } (2, 1)] \approx 0.000; +$
$\Pr[\text{class } (2, 2)] \approx 0.184; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

  Negate $a_{S,T}$

    if S contains collision.

  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

    For each T:

      Diffuse $a_{S,T}$ across all S.

    For each S:

      Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

8 negations and 368 diffusions:

Pr[class $(0, 0)$] $\approx 0.708; +$

Pr[class $(0, 1)$] $\approx 0.003; +$

Pr[class $(1, 0)$] $\approx 0.001; -$

Pr[class $(1, 1)$] $\approx 0.046; +$

Pr[class $(1, 2)$] $\approx 0.007; +$

Pr[class $(2, 1)$] $\approx 0.000; +$

Pr[class $(2, 2)$] $\approx 0.234; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

  Negate $a_{S,T}$

    if S contains collision.

  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

    For each T:

      Diffuse $a_{S,T}$ across all S.

    For each S:

      Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

9 negations and 414 diffusions:

Pr[class $(0, 0)$] $\approx 0.658; +$

Pr[class $(0, 1)$] $\approx 0.003; +$

Pr[class $(1, 0)$] $\approx 0.001; -$

Pr[class $(1, 1)$] $\approx 0.042; +$

Pr[class $(1, 2)$] $\approx 0.009; +$

Pr[class $(2, 1)$] $\approx 0.000; +$

Pr[class $(2, 2)$] $\approx 0.287; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

  Negate $a_{S,T}$

    if S contains collision.

  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

    For each T:

      Diffuse $a_{S,T}$ across all S.

    For each S:

      Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

10 negations and 460 diffusions:

Pr[class $(0, 0)$] $\approx 0.606; +$

Pr[class $(0, 1)$] $\approx 0.003; +$

Pr[class $(1, 0)$] $\approx 0.002; -$

Pr[class $(1, 1)$] $\approx 0.037; +$

Pr[class $(1, 2)$] $\approx 0.013; +$

Pr[class $(2, 1)$] $\approx 0.000; +$

Pr[class $(2, 2)$] $\approx 0.338; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
Negate $a_{S,T}$
if S contains collision.
Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
For each T:
Diffuse $a_{S,T}$ across all S.
For each S:
Diffuse $a_{S,T}$ across all T.

Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
11 negations and 506 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.547; +$
$\Pr[\text{class } (0,1)] \approx 0.004; +$
$\Pr[\text{class } (1,0)] \approx 0.003; -$
$\Pr[\text{class } (1,1)] \approx 0.036; +$
$\Pr[\text{class } (1,2)] \approx 0.015; +$
$\Pr[\text{class } (2,1)] \approx 0.001; +$
$\Pr[\text{class } (2,2)] \approx 0.394; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
Negate $a_{S,T}$
if S contains collision.
Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
For each T:
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For each S:
Diffuse $a_{S,T}$ across all T.

Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
12 negations and 552 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.491; +$
$\Pr[\text{class } (0, 1)] \approx 0.004; +$
$\Pr[\text{class } (1, 0)] \approx 0.003; -$
$\Pr[\text{class } (1, 1)] \approx 0.032; +$
$\Pr[\text{class } (1, 2)] \approx 0.014; +$
$\Pr[\text{class } (2, 1)] \approx 0.001; +$
$\Pr[\text{class } (2, 2)] \approx 0.455; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
  Negate $a_{S,T}$
    if S contains collision.
  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
    For each T:
      Diffuse $a_{S,T}$ across all S.
    For each S:
      Diffuse $a_{S,T}$ across all T.

Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
13 negations and 598 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.436; +$
$\Pr[\text{class } (0,1)] \approx 0.005; +$
$\Pr[\text{class } (1,0)] \approx 0.003; -$
$\Pr[\text{class } (1,1)] \approx 0.026; +$
$\Pr[\text{class } (1,2)] \approx 0.017; +$
$\Pr[\text{class } (2,1)] \approx 0.000; +$
$\Pr[\text{class } (2,2)] \approx 0.513; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

14 negations and 644 diffusions:

Pr[class $(0, 0)$] $\approx 0.377; +$

Pr[class $(0, 1)$] $\approx 0.006; +$

Pr[class $(1, 0)$] $\approx 0.004; -$

Pr[class $(1, 1)$] $\approx 0.025; +$

Pr[class $(1, 2)$] $\approx 0.022; +$

Pr[class $(2, 1)$] $\approx 0.001; +$

Pr[class $(2, 2)$] $\approx 0.566; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

15 negations and 690 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.322; +$

$\Pr[\text{class } (0, 1)] \approx 0.005; +$

$\Pr[\text{class } (1, 0)] \approx 0.004; -$

$\Pr[\text{class } (1, 1)] \approx 0.021; +$

$\Pr[\text{class } (1, 2)] \approx 0.023; +$

$\Pr[\text{class } (2, 1)] \approx 0.001; +$

$\Pr[\text{class } (2, 2)] \approx 0.623; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

16 negations and 736 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.270; +$

$\Pr[\text{class } (0,1)] \approx 0.006; +$

$\Pr[\text{class } (1,0)] \approx 0.005; -$

$\Pr[\text{class } (1,1)] \approx 0.017; +$

$\Pr[\text{class } (1,2)] \approx 0.022; +$

$\Pr[\text{class } (2,1)] \approx 0.001; +$

$\Pr[\text{class } (2,2)] \approx 0.680; +$

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How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
  Negate $a_{S,T}$
    if S contains collision.
  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
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Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
17 negations and 782 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.218; +$
$\Pr[\text{class } (0, 1)] \approx 0.007; +$
$\Pr[\text{class } (1, 0)] \approx 0.005; -$
$\Pr[\text{class } (1, 1)] \approx 0.015; +$
$\Pr[\text{class } (1, 2)] \approx 0.024; +$
$\Pr[\text{class } (2, 1)] \approx 0.001; +$
$\Pr[\text{class } (2, 2)] \approx 0.730; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

  Negate $a_{S,T}$

    if S contains collision.

  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

    For each T:

      Diffuse $a_{S,T}$ across all S.

    For each S:

      Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

18 negations and 828 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.172; +$

$\Pr[\text{class } (0,1)] \approx 0.006; +$

$\Pr[\text{class } (1,0)] \approx 0.005; -$

$\Pr[\text{class } (1,1)] \approx 0.011; +$

$\Pr[\text{class } (1,2)] \approx 0.029; +$

$\Pr[\text{class } (2,1)] \approx 0.001; +$

$\Pr[\text{class } (2,2)] \approx 0.775; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
Negate $a_{S,T}$
if S contains collision.
Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
For each T:
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Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
19 negations and 874 diffusions:

Pr[class $(0, 0)$] ≈ 0.131; $+$
Pr[class $(0, 1)$] ≈ 0.007; $+$
Pr[class $(1, 0)$] ≈ 0.006; $-$
Pr[class $(1, 1)$] ≈ 0.008; $+$
Pr[class $(1, 2)$] ≈ 0.030; $+$
Pr[class $(2, 1)$] ≈ 0.002; $+$
Pr[class $(2, 2)$] ≈ 0.816; $+$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

20 negations and 920 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.093; +$

$\Pr[\text{class } (0,1)] \approx 0.007; +$

$\Pr[\text{class } (1,0)] \approx 0.007; -$

$\Pr[\text{class } (1,1)] \approx 0.007; +$

$\Pr[\text{class } (1,2)] \approx 0.027; +$

$\Pr[\text{class } (2,1)] \approx 0.002; +$

$\Pr[\text{class } (2,2)] \approx 0.857; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

    Negate $a_{S,T}$

        if S contains collision.

    Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

        For each T:

            Diffuse $a_{S,T}$ across all S.

        For each S:

            Diffuse $a_{S,T}$ across all T.

Now high probability
that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
21 negations and 966 diffusions:

$\Pr[\text{class } (0, 0)] \approx 0.062; +$
$\Pr[\text{class } (0, 1)] \approx 0.007; +$
$\Pr[\text{class } (1, 0)] \approx 0.006; -$
$\Pr[\text{class } (1, 1)] \approx 0.004; +$
$\Pr[\text{class } (1, 2)] \approx 0.030; +$
$\Pr[\text{class } (2, 1)] \approx 0.001; +$
$\Pr[\text{class } (2, 2)] \approx 0.890; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

22 negations and 1012 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.037; +$

$\Pr[\text{class } (0,1)] \approx 0.008; +$

$\Pr[\text{class } (1,0)] \approx 0.007; -$

$\Pr[\text{class } (1,1)] \approx 0.002; +$

$\Pr[\text{class } (1,2)] \approx 0.034; +$

$\Pr[\text{class } (2,1)] \approx 0.001; +$

$\Pr[\text{class } (2,2)] \approx 0.910; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

23 negations and 1058 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.017; +$

$\Pr[\text{class } (0,1)] \approx 0.008; +$

$\Pr[\text{class } (1,0)] \approx 0.007; -$

$\Pr[\text{class } (1,1)] \approx 0.002; +$

$\Pr[\text{class } (1,2)] \approx 0.034; +$

$\Pr[\text{class } (2,1)] \approx 0.002; +$

$\Pr[\text{class } (2,2)] \approx 0.930; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
  Negate $a_{S,T}$
    if S contains collision.
  Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
    For each T:
      Diffuse $a_{S,T}$ across all S.
    For each S:
      Diffuse $a_{S,T}$ across all T.

Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
24 negations and 1104 diffusions:

Pr[class $(0, 0)$] $\approx 0.005; +$
Pr[class $(0, 1)$] $\approx 0.007; +$
Pr[class $(1, 0)$] $\approx 0.007; -$
Pr[class $(1, 1)$] $\approx 0.000; +$
Pr[class $(1, 2)$] $\approx 0.030; +$
Pr[class $(2, 1)$] $\approx 0.002; +$
Pr[class $(2, 2)$] $\approx 0.948; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

25 negations and 1150 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.000; +$

$\Pr[\text{class } (0,1)] \approx 0.008; +$

$\Pr[\text{class } (1,0)] \approx 0.008; -$

$\Pr[\text{class } (1,1)] \approx 0.000; +$

$\Pr[\text{class } (1,2)] \approx 0.031; +$

$\Pr[\text{class } (2,1)] \approx 0.001; +$

$\Pr[\text{class } (2,2)] \approx 0.952; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.
Repeat $\approx 0.6 \cdot 2^n/r$ times:
Negate $a_{S,T}$
if S contains collision.
Repeat $\approx 0.7 \cdot \sqrt{r}$ times:
For each T:
Diffuse $a_{S,T}$ across all S.
For each S:
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Now high probability
that T contains collision.
Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to
$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;
reduce a to low-dim vector.
Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after
26 negations and 1196 diffusions:

$\Pr[\text{class } (0,0)] \approx 0.002; -$
$\Pr[\text{class } (0,1)] \approx 0.008; +$
$\Pr[\text{class } (1,0)] \approx 0.008; -$
$\Pr[\text{class } (1,1)] \approx 0.000; -$
$\Pr[\text{class } (1,2)] \approx 0.035; +$
$\Pr[\text{class } (2,1)] \approx 0.002; +$
$\Pr[\text{class } (2,2)] \approx 0.945; +$

Right column is sign of $a_{S,T}$.

How the quantum walk works:

Start from uniform superposition.

Repeat $\approx 0.6 \cdot 2^n/r$ times:

Negate $a_{S,T}$

if S contains collision.

Repeat $\approx 0.7 \cdot \sqrt{r}$ times:

For each T:

Diffuse $a_{S,T}$ across all S.

For each S:

Diffuse $a_{S,T}$ across all T.

Now high probability

that T contains collision.

Cost $r + 2^n/\sqrt{r}$. Optimize: $2^{2n/3}$.

Classify (S, T) according to

$(\#(S \cap \{p, q\}), \#(T \cap \{p, q\}))$;

reduce a to low-dim vector.

Analyze evolution of this vector.

e.g. $n = 15$, $r = 1024$, after

27 negations and 1242 diffusions:

Pr[class $(0, 0)$] $\approx 0.011; -$

Pr[class $(0, 1)$] $\approx 0.007; +$

Pr[class $(1, 0)$] $\approx 0.007; -$

Pr[class $(1, 1)$] $\approx 0.001; -$

Pr[class $(1, 2)$] $\approx 0.034; +$

Pr[class $(2, 1)$] $\approx 0.003; +$

Pr[class $(2, 2)$] $\approx 0.938; +$

Right column is sign of $a_{S,T}$.

e quantum walk works:

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$\Pr[\text{class } (2, 2)] \approx 0.938; +$

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Optimize: $2^{2n/3}$.

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Data structures

Moving from $D(S$
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Data structures

Moving from $D(S)$ to $D(T)$ dominated by $O(1)$ evaluati of f if f is extremely slow.

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Data structures

Moving from $D(S)$ to $D(T)$: dominated by $O(1)$ evaluations of f if f is extremely slow.

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Data structures

Moving from $D(S)$ to $D(T)$: dominated by $O(1)$ evaluations of f if f is extremely slow.

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Data structures

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Moving from $D(S)$ to $D(T)$: dominated by $O(1)$ evaluations of f if f is extremely slow.

But usually f is not so slow. Store set S and multiset $f(S)$ in, e.g., hash tables?

Minor problem: time to hash S is huge for some sets S.

Fix: randomize hash function (1979 Carter–Wegman), and specify big enough time for whole algorithm to be reliable.

Major problem: hash table depends on history, not just on S. Algorithm fails horribly.

Need history-independent $D(S)$.

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Quantum computer type 3 (QC3):
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General belief: any QC2 is a QC3.
Argument for belief:
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A note on D-Wave

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$2, 7, -1, 8, 1, -8, -2, 8)$.
$0, 0, 0, 0, 1, 0, 0)$.

ored in 4 qubits: a list of
bers, not all zero. e.g.:
$1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3)$.

ored in 64 qubits:
2^{64} numbers, not all zero.

ored in 1000 qubits: a list
numbers, not all zero.

Measuring a quantum computer

Can simply look at a bit.
Cannot simply look at the list
of numbers stored in n qubits.

Measuring n qubits
- produces n bits and
- destroys the state.

If n qubits have state
$(a_0, a_1, \ldots, a_{2^n-1})$ then
measurement produces q
with probability $|a_q|^2 / \sum_r |a_r|^2$.

State is then all zeros
except 1 at position q.

e.g.: Say
$(1, 1, 1,$

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ubits:

s, not all zero.

$9, 2, 6)$.

$, 1, -8, -2, 8)$.

$1, 0, 0)$.

ubits: a list of

ll zero. e.g.:

$, 5, 3, 5, 8, 9, 7, 9, 3)$.

qubits:

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00 qubits: a list

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e.g.: Say 3 qubits

$(1, 1, 1, 1, 1, 1, 1, 1$

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"Quantum RNG."

Measuring a quantum computer

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"Quantum RNG."

Warning: Quantum RNGs sold today are measurably biased.

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e.g.: Say
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e.g.: Say 3 qubits
$(3, 1, 4, 1, 5, 9, 2, 6$

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Measurement produces
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e.g.: Say 3 qubits have state $(3, 1, 4, 1, 5, 9, 2, 6)$.

Measurement produces
$000 = 0$ with probability $9/173$;
$001 = 1$ with probability $1/173$;
$010 = 2$ with probability $16/173$;
$011 = 3$ with probability $1/173$;
$100 = 4$ with probability $25/173$;
$101 = 5$ with probability $81/173$;
$110 = 6$ with probability $4/173$;
$111 = 7$ with probability $36/173$.

e.g.: Say 3 qubits have state $(1, 1, 1, 1, 1, 1, 1, 1)$.

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5 is most likely outcome.

y 3 qubits have state
$1, 1, 1, 1, 1)$.

ement produces
with probability $1/8$;
with probability $1/8$;
with probability $1/8$;
with probability $1/8$;
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$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

5 is guaranteed outcome.

y 3 qubits have state
1, 5, 9, 2, 6).

ement produces
with probability 9/173;
with probability 1/173;
with probability 16/173;
with probability 1/173;
with probability 25/173;
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with probability 4/173;
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5 is guaranteed outcome.

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$(3, 1, 4, 1$
$(1, 3, 1, 4$

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ability $9/173$;

ability $1/173$;

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ability $1/173$;

ability $25/173$;

ability $81/173$;

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ability $36/173$.

tcome.

e.g.: Say 3 qubits have state

$(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces

$000 = 0$ with probability 0;

$001 = 1$ with probability 0;

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$011 = 3$ with probability 0;

$100 = 4$ with probability 0;

$101 = 5$ with probability 1;

$110 = 6$ with probability 0;

$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 q

$(3, 1, 4, 1, 5, 9, 2, 6$

$(1, 3, 1, 4, 9, 5, 6, 2$

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e.g.: Say 3 qubits have state $(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces
$000 = 0$ with probability 0;
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5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 qubits: $(3, 1, 4, 1, 5, 9, 2, 6) \mapsto (1, 3, 1, 4, 9, 5, 6, 2)$.

e.g.: Say 3 qubits have state $(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces
$000 = 0$ with probability 0;
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NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
$(1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

e.g.: Say 3 qubits have state $(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces
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NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

e.g.: Say 3 qubits have state
$(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
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$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

NOT_2 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

y 3 qubits have state
0, 0, 1, 0, 0).

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with probability 0;
with probability 0;
with probability 0;
with probability 0;
with probability 0;
with probability 1;
with probability 0;
with probability 0.

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NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

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NOT gates

NOT_0 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:

$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$

$(1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

NOT_1 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(4, 1, 3, 1, 2, 6, 5, 9)$.

NOT_2 gate on 3 qubits:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(5, 9, 2, 6, 3, 1, 4, 1)$.

state

$(1, 0, 0, 0, 0, 0, 0, 0$

$(0, 1, 0, 0, 0, 0, 0, 0$

$(0, 0, 1, 0, 0, 0, 0, 0$

$(0, 0, 0, 1, 0, 0, 0, 0$

$(0, 0, 0, 0, 1, 0, 0, 0$

$(0, 0, 0, 0, 0, 1, 0, 0$

$(0, 0, 0, 0, 0, 0, 1, 0$

$(0, 0, 0, 0, 0, 0, 0, 1$

Operation on quan

NOT_0, swapping p

Operation after m

flipping bit 0 of re

Flip: output is not

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NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
$(1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

NOT_2 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measure
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state
NOT_0, swapping pairs.
Operation after measuremen
flipping bit 0 of result.
Flip: output is not input.

NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
$(1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

NOT_2 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

tes

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$4, 9, 5, 6, 2).$

ate on 4 qubits:

$5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$

$9,5,6,2,3,5,8,5,7,9,3,9).$

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$1, 2, 6, 5, 9).$

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$6, 3, 1, 4, 1).$

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:

NOT_0, swapping pairs.

Operation after measurement:

flipping bit 0 of result.

Flip: output is not input.

Controll

e.g. CNO

$(3, 1, 4, 1$

$(3, 1, 1, 4$

ubits:

$) \mapsto$

$).$

ubits:

$3,5,8,9,7,9,3) \mapsto$

$5,8,5,7,9,3,9).$

ubits:

$) \mapsto$

$).$

ubits:

$) \mapsto$

$).$

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:

NOT_0, swapping pairs.

Operation after measurement:

flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT g

e.g. $\mathrm{CNOT}_{1,0}$:

$(3,1,4,1,5,9,2,6$

$(3,1,1,4,5,9,6,2$

$,3) \mapsto$
$,9).$

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
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Controlled-NOT gates

e.g. $\mathrm{CNOT}_{1,0}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
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$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
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Operation after measurement:
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Flip: output is not input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.

Operation after measurement:
flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state: NOT_0, swapping pairs.

Operation after measurement: flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT gates

e.g. $\mathrm{CNOT}_{1,0}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement: flipping bit 0 *if* bit 1 is set; i.e., $(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\mathrm{CNOT}_{2,0}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. $\mathrm{CNOT}_{0,2}$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

state	measurement
0, 0, 0, 0, 0)	000
0, 0, 0, 0, 0)	001
0, 0, 0, 0, 0)	010
1, 0, 0, 0, 0)	011
0, 1, 0, 0, 0)	100
0, 0, 1, 0, 0)	101
0, 0, 0, 1, 0)	110
0, 0, 0, 0, 1)	111

on on quantum state:
swapping pairs.
on after measurement:
bit 0 of result.
tput is not input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. $\text{CNOT}_{0,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli g

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controlle

e.g. CCN
(3, 1, 4, 1
(3, 1, 4, 1

measurement

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0)	001
0)	010
0)	011
0)	100
0)	101
0)	110
1)	111

ntum state:

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t input.

Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. $\text{CNOT}_{0,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as
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e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6$
$(3, 1, 4, 1, 5, 9, 6, 2$

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Controlled-NOT gates

e.g. $\text{CNOT}_{1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
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e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as
controlled-controlled-NOT g

e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Controlled-NOT gates

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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Toffoli gates

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Controlled-NOT gates

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

Controlled-NOT gates

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
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e.g. $\text{CNOT}_{2,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. $\text{CNOT}_{0,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

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controlled-controlled-NOT gates.

e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $\text{CCNOT}_{0,1,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

ed-NOT gates

OT$_{1,0}$:

$1, 5, 9, 2, 6) \mapsto$

$4, 5, 9, 6, 2)$.

on after measurement:

bit 0 *if* bit 1 is set; i.e.,

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

OT$_{2,0}$:

$1, 5, 9, 2, 6) \mapsto$

$1, 9, 5, 6, 2)$.

OT$_{0,2}$:

$1, 5, 9, 2, 6) \mapsto$

$6, 5, 1, 2, 1)$.

Toffoli gates

Also known as controlled-controlled-NOT gates.

e.g. CCNOT$_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. CCNOT$_{0,1,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

More sh

Combine

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ates

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t 1 is set; i.e.,

, $q_1, q_0 \oplus q_1$).

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Toffoli gates

Also known as controlled-controlled-NOT gates.

e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $\text{CCNOT}_{0,1,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

More shuffling

Combine NOT, CN

to build other perr

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i.e.,

q_1).

Toffoli gates

Also known as
controlled-controlled-NOT gates.

e.g. $\text{CCNOT}_{2,1,0}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $\text{CCNOT}_{0,1,2}$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

More shuffling

Combine NOT, CNOT, Toff
to build other permutations.

Toffoli gates

Also known as
controlled-controlled-NOT gates.

e.g. $\text{CCNOT}_{2,1,0}$:
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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

More shuffling

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Toffoli gates

Also known as controlled-controlled-NOT gates.

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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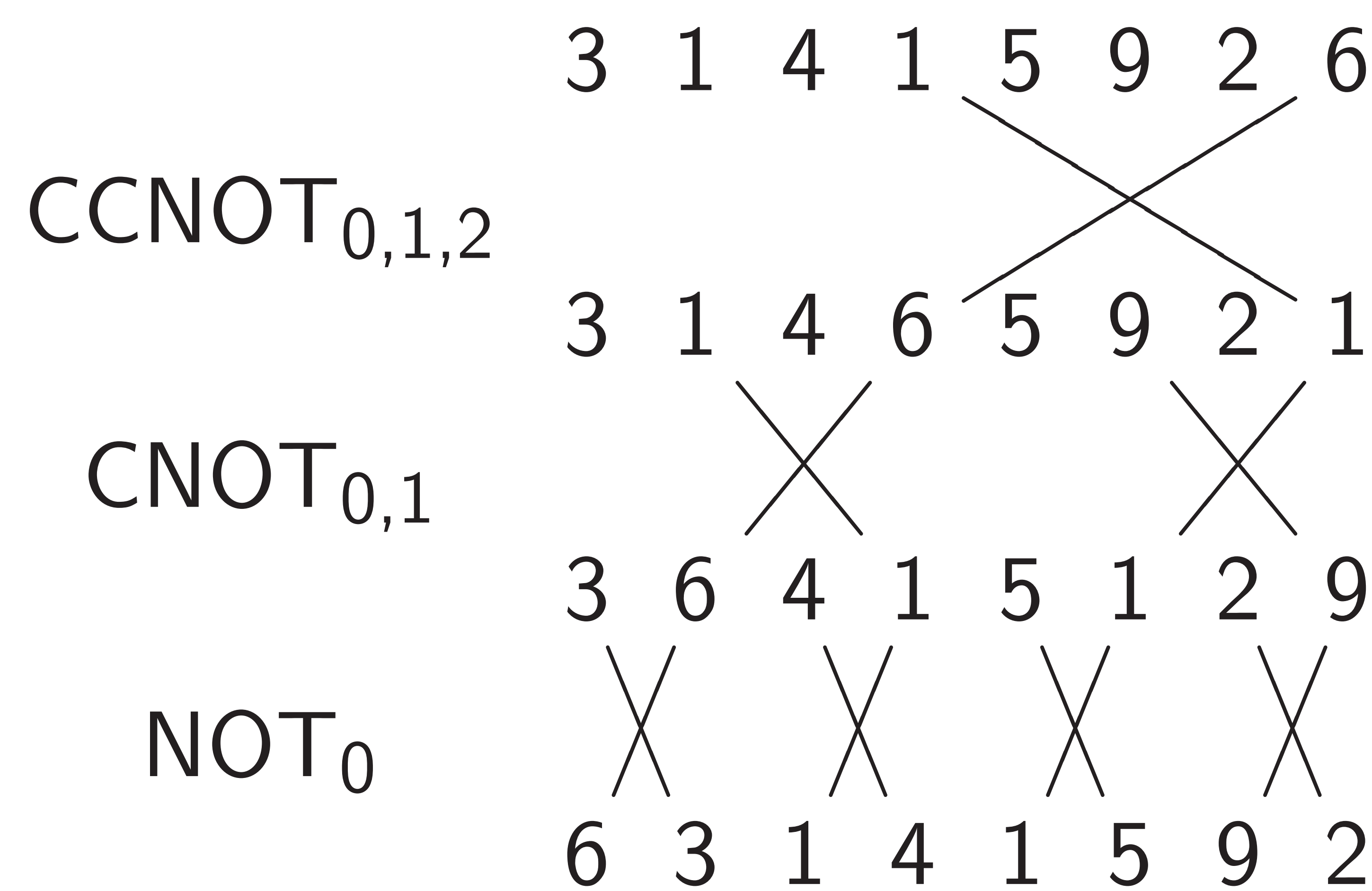
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:



ates

own as

ed-controlled-NOT gates.

$\text{NOT}_{2,1,0}$:

$1, 5, 9, 2, 6) \mapsto$

$1, 5, 9, 6, 2)$.

on after measurement:

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

$\text{NOT}_{0,1,2}$:

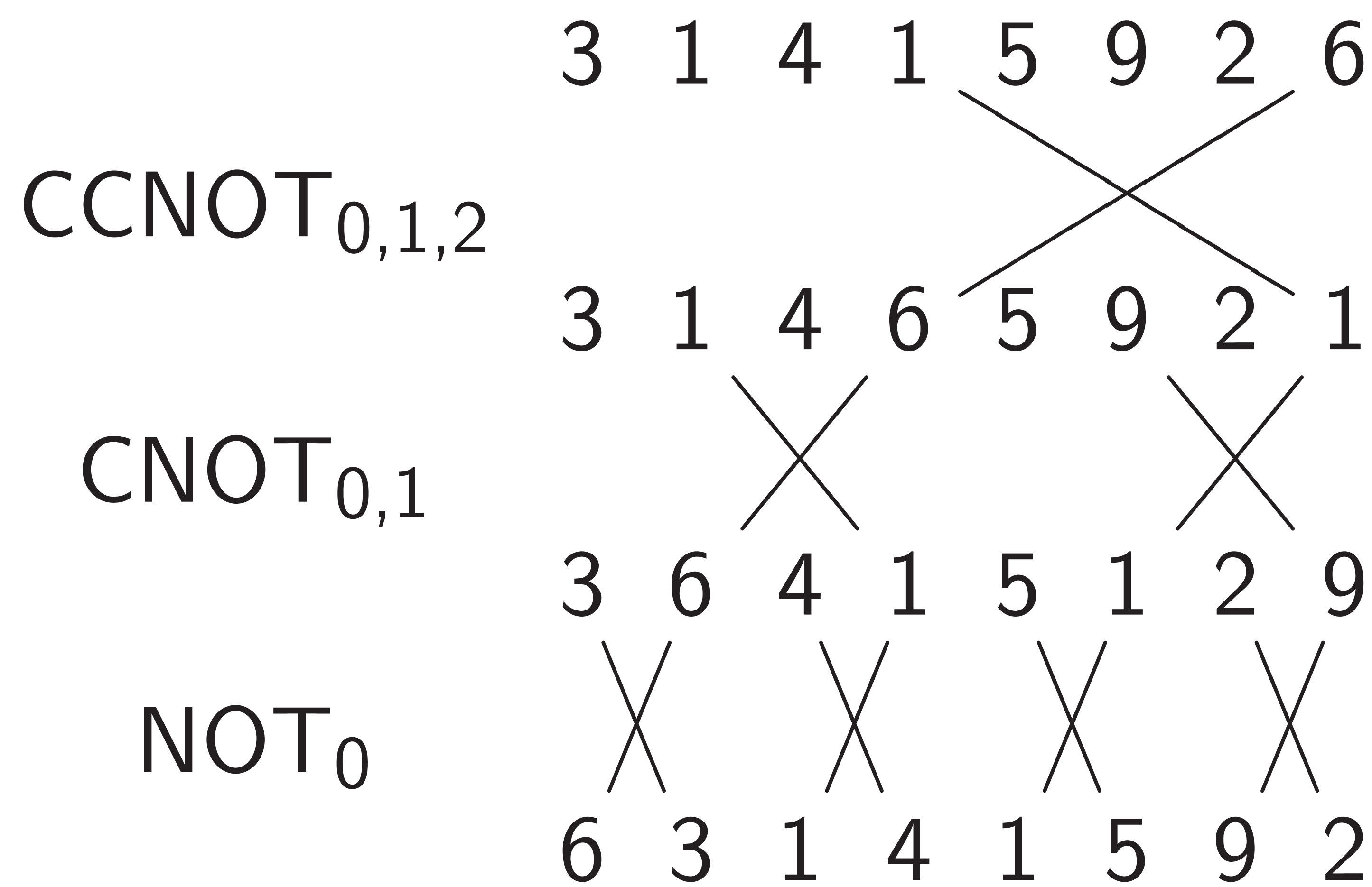
$1, 5, 9, 2, 6) \mapsto$

$5, 5, 9, 2, 1)$.

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

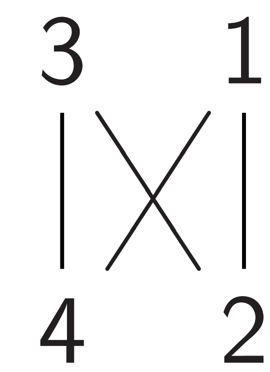
e.g. series of gates to rotate 8 positions by distance 1:



Hadama

Hadama

$(a, b) \mapsto$



ed-NOT gates.

$) \mapsto$

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easurement:

$, q_1, q_0 \oplus q_1 q_2).$

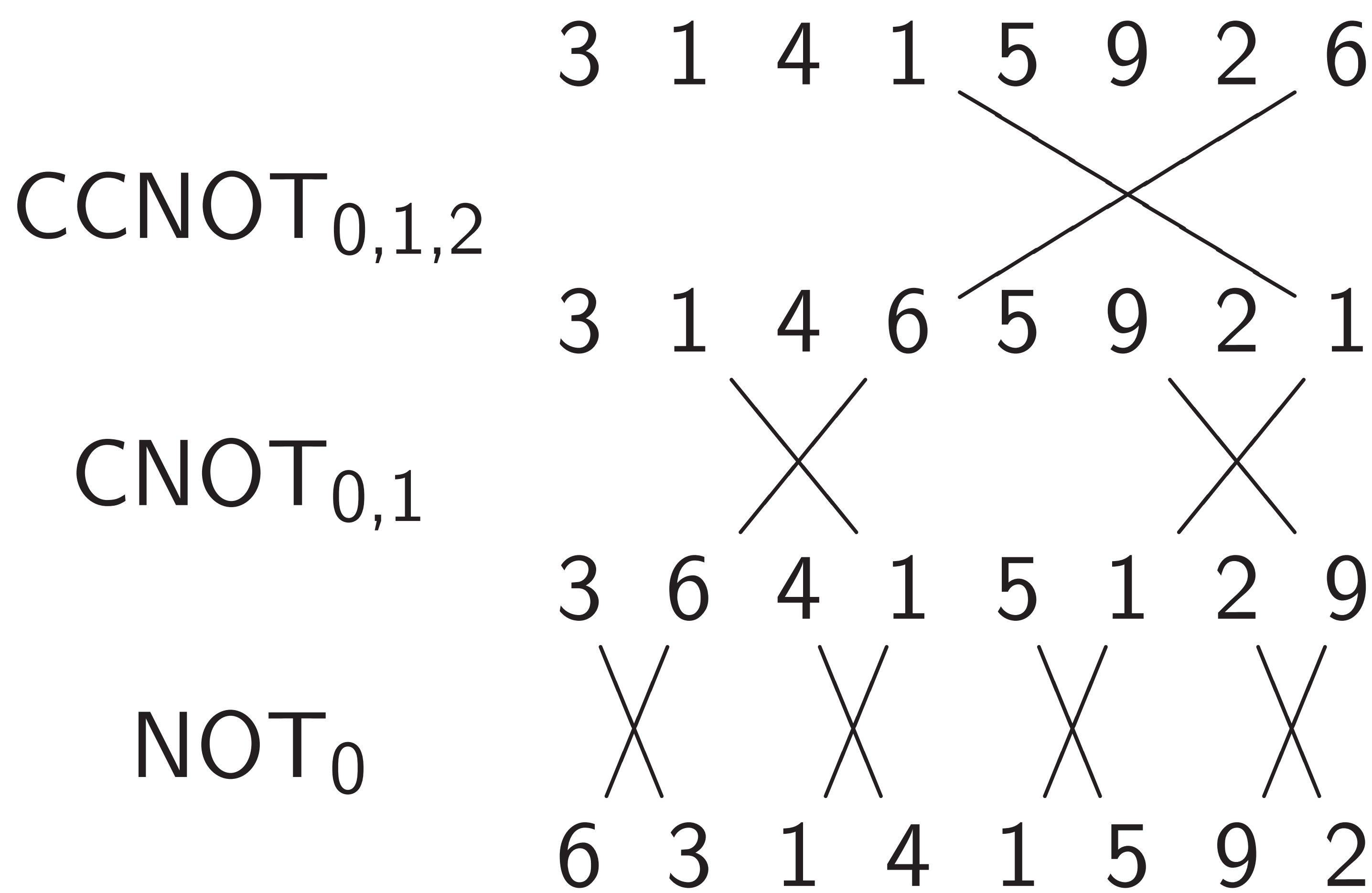
$) \mapsto$

$).$

More shuffling

Combine NOT, CNOT, Toffoli
to build other permutations.

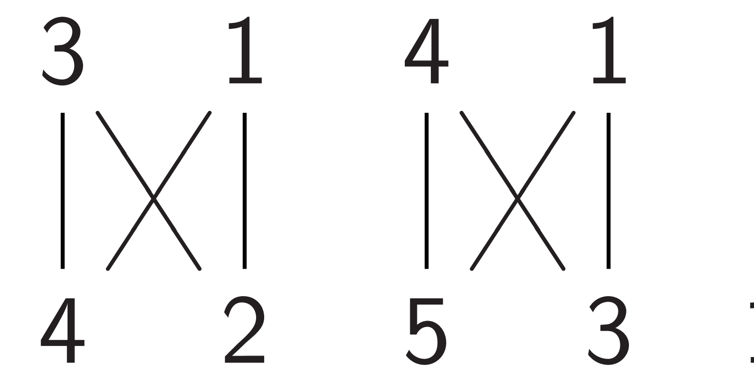
e.g. series of gates to
rotate 8 positions by distance 1:



Hadamard gates

Hadamard$_0$:

$(a, b) \mapsto (a + b, a$



ates.

nt:

$q_1 q_2)$.

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:

3 1 4 1 5 9 2 6

$\text{CCNOT}_{0,1,2}$

3 1 4 6 5 9 2 1

$\text{CNOT}_{0,1}$

3 6 4 1 5 1 2 9

NOT_0

6 3 1 4 1 5 9 2

Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.

3 1 4 1 5 9 2

4 2 5 3 14 −4 8

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:

3 1 4 1 5 9 2 6

$\text{CCNOT}_{0,1,2}$

3 1 4 6 5 9 2 1

$\text{CNOT}_{0,1}$

3 6 4 1 5 1 2 9

NOT_0

6 3 1 4 1 5 9 2

Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.

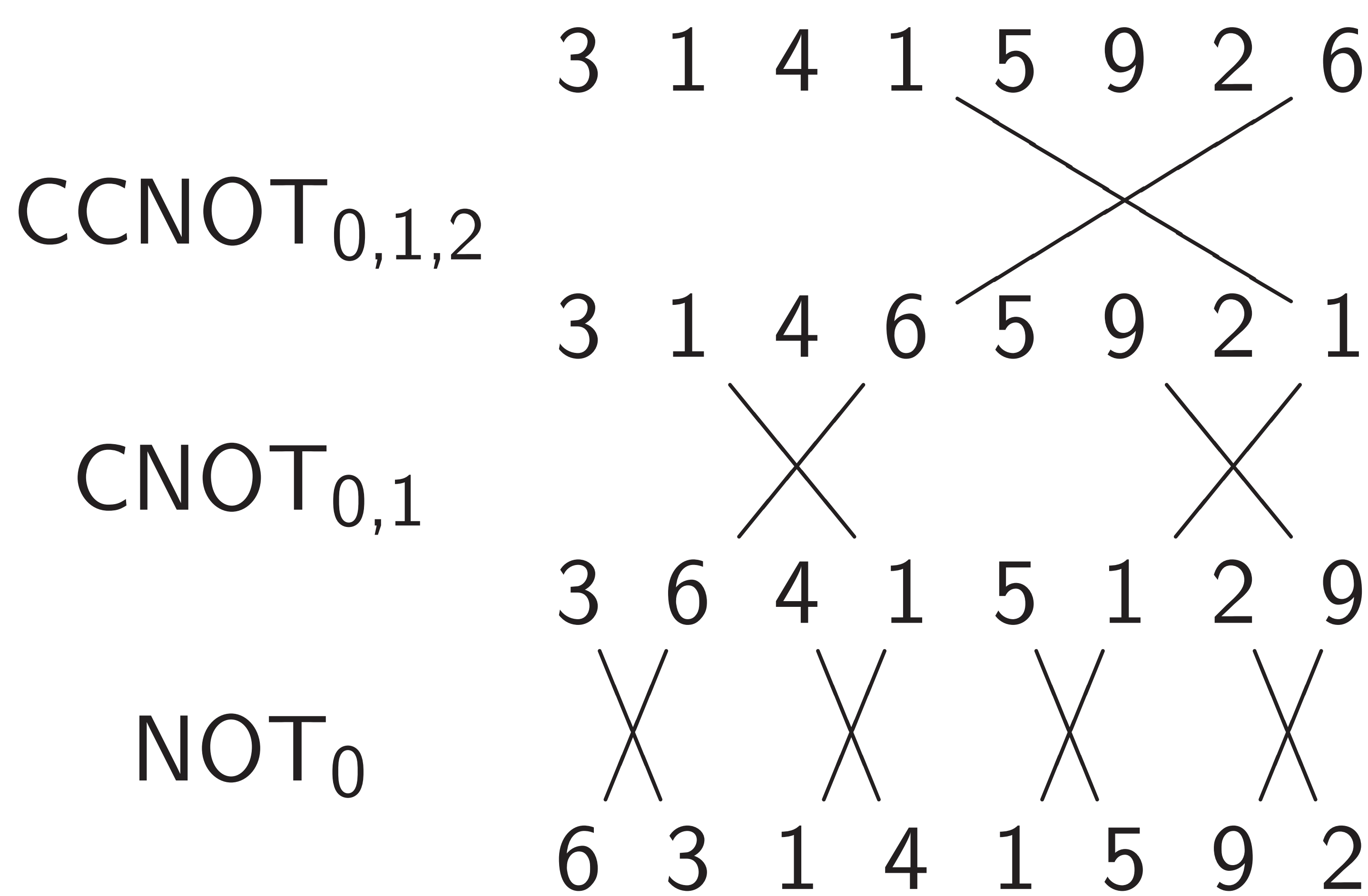
3 1 4 1 5 9 2 6

4 2 5 3 14 −4 8 −4

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

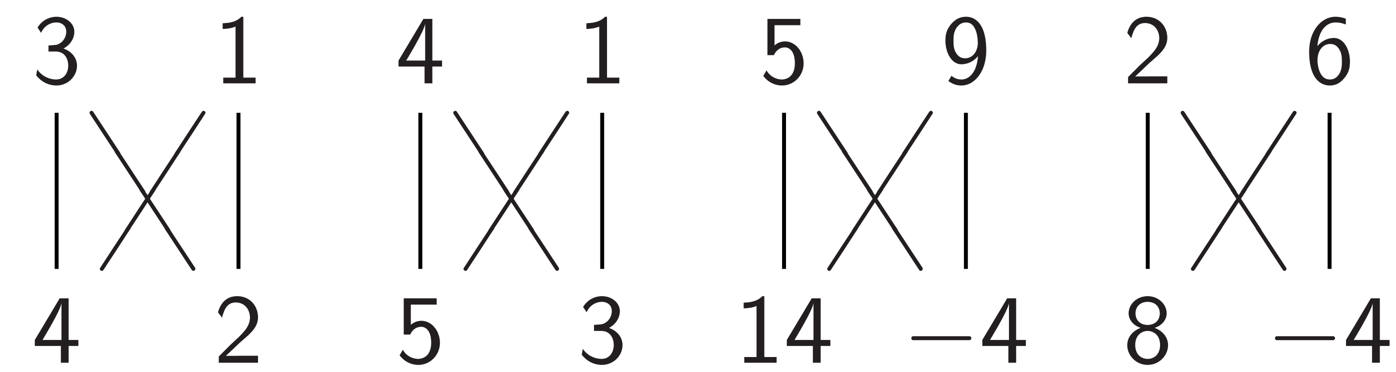
e.g. series of gates to rotate 8 positions by distance 1:



Hadamard gates

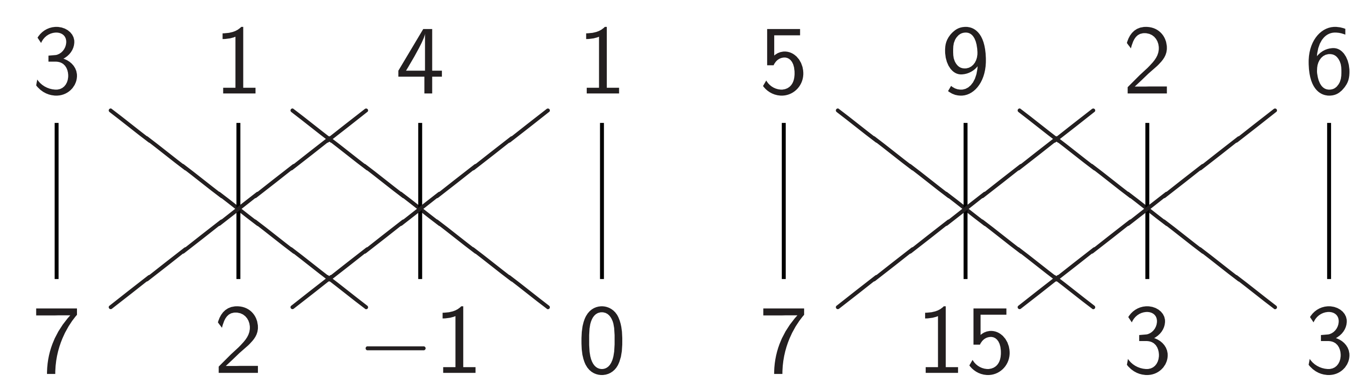
$\mathrm{Hadamard}_0$:

$(a, b) \mapsto (a + b, a - b)$.



$\mathrm{Hadamard}_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



uffling

e NOT, CNOT, Toffoli
other permutations.

es of gates to
positions by distance 1:

3 1 4 1 5 9 2 6

$_{0,1,2}$

3 1 4 6 5 9 2 1

$_{0,1}$

3 6 4 1 5 1 2 9

$_{0}$

6 3 1 4 1 5 9 2

Hadamard gates

Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.

3 1 4 1 5 9 2 6

4 2 5 3 14 −4 8 −4

Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.

3 1 4 1 5 9 2 6

7 2 −1 0 7 15 3 3

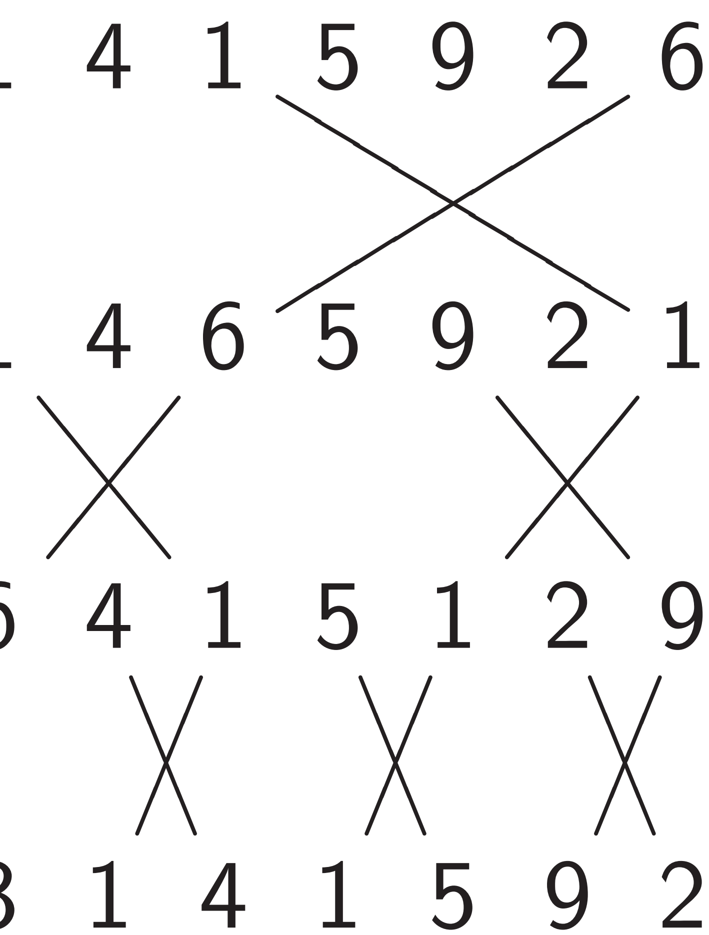
Simon's

Step 1.

1, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0
0, 0, 0, 0

NOT, Toffoli
mutations.

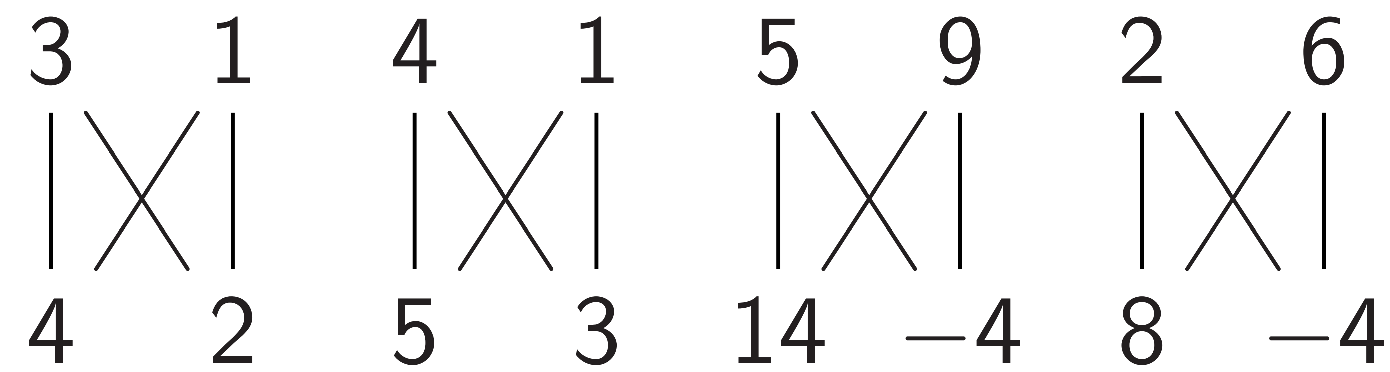
s to
by distance 1:



Hadamard gates

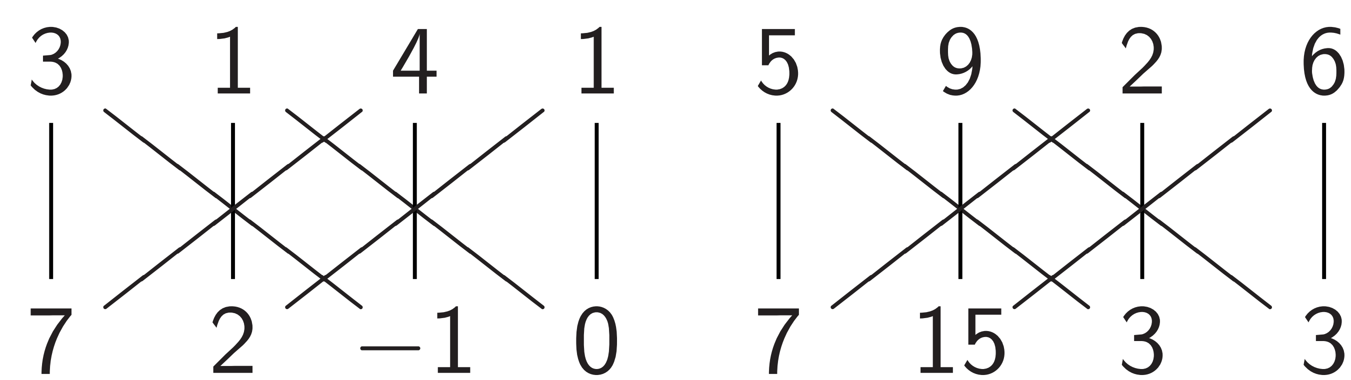
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 1. Set up pu

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

foli

ce 1:

9 2 6

9 2 1

1 2 9

5 9 2

Hadamard gates

Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.

$$\begin{array}{cccccccc} 3 & 1 & 4 & 1 & 5 & 9 & 2 & 6 \\ 4 & 2 & 5 & 3 & 14 & -4 & 8 & -4 \end{array}$$

Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.

$$\begin{array}{cccccccc} 3 & 1 & 4 & 1 & 5 & 9 & 2 & 6 \\ 7 & 2 & -1 & 0 & 7 & 15 & 3 & 3 \end{array}$$

Simon's algorithm

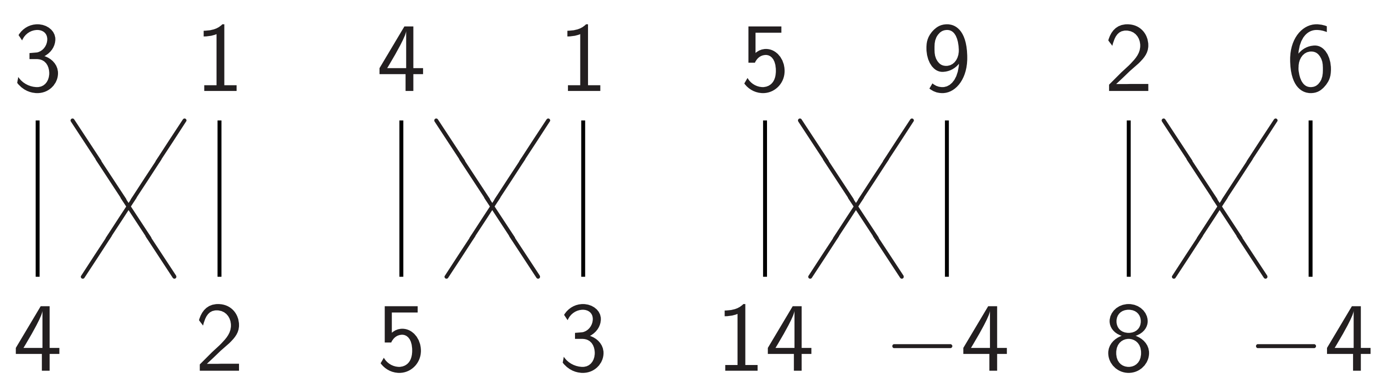
Step 1. Set up pure zero sta

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

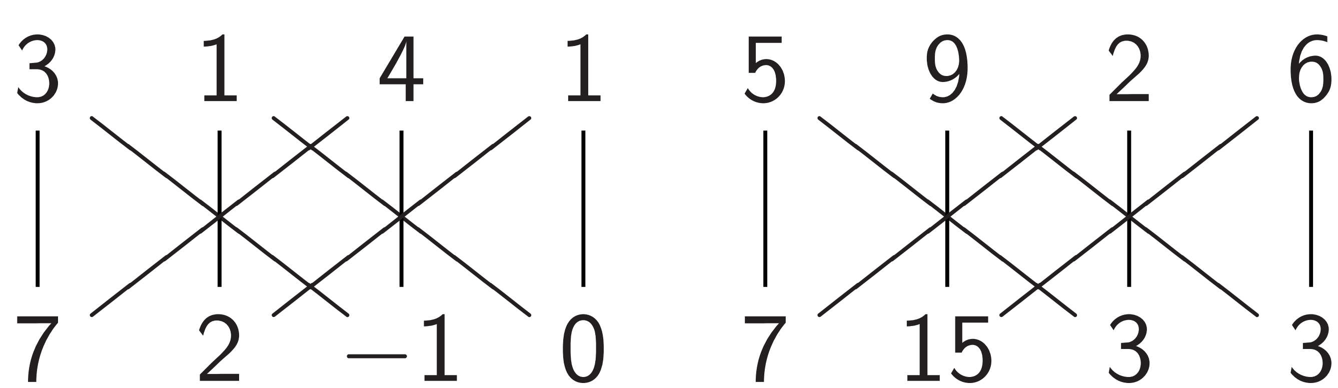
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

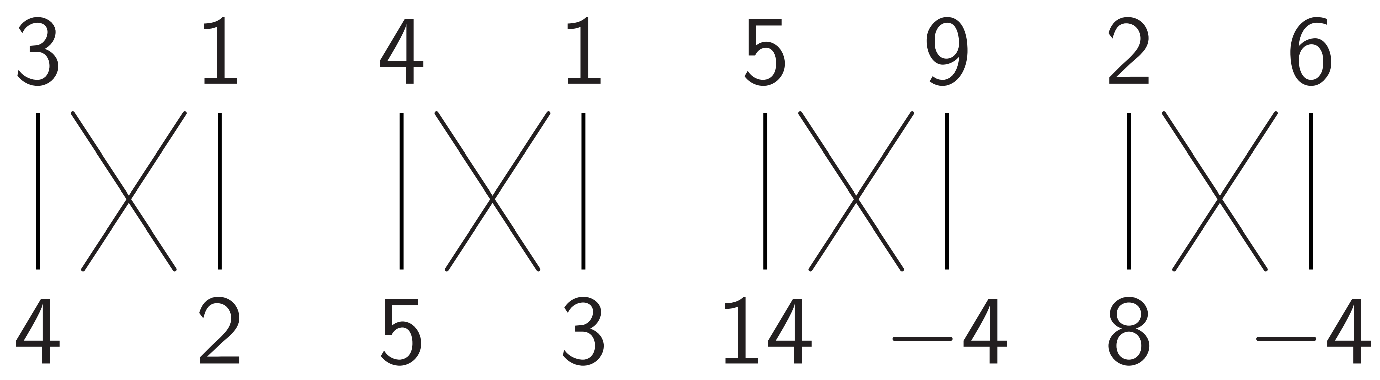
Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

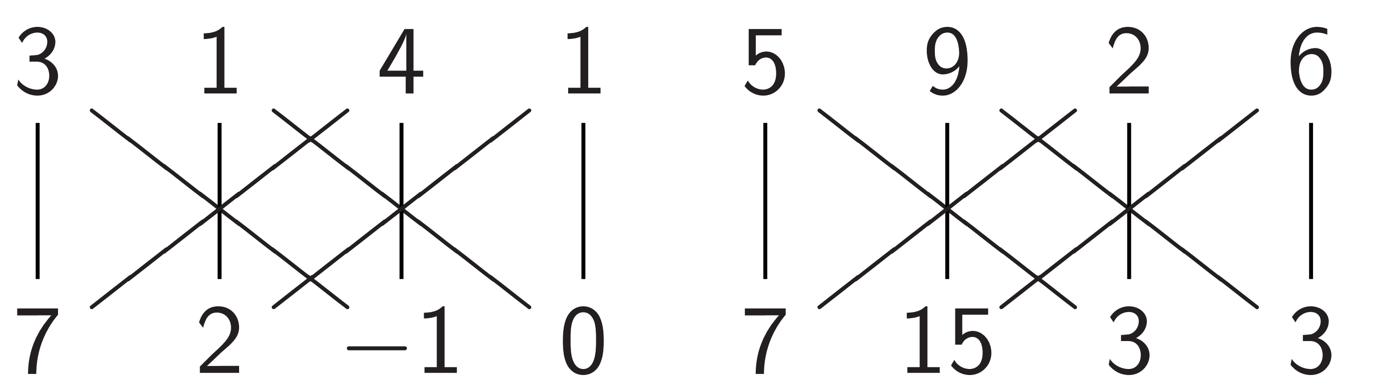
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 2. Hadamard$_0$:

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.

$$\begin{array}{cccccccc} 3 & 1 & 4 & 1 & 5 & 9 & 2 & 6 \\ 4 & 2 & 5 & 3 & 14 & -4 & 8 & -4 \end{array}$$

Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.

$$\begin{array}{cccccccc} 3 & 1 & 4 & 1 & 5 & 9 & 2 & 6 \\ 7 & 2 & -1 & 0 & 7 & 15 & 3 & 3 \end{array}$$

Simon's algorithm

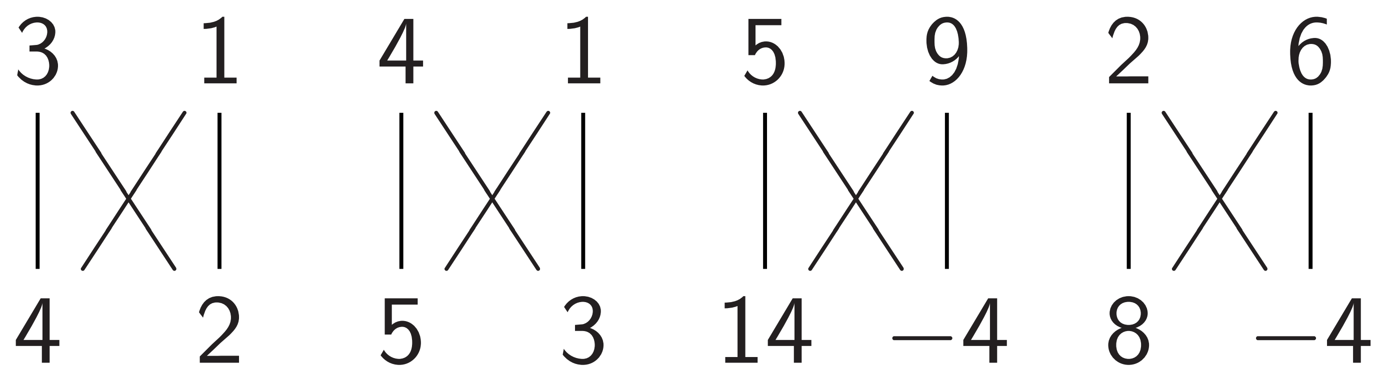
Step 3. Hadamard$_1$:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Hadamard gates

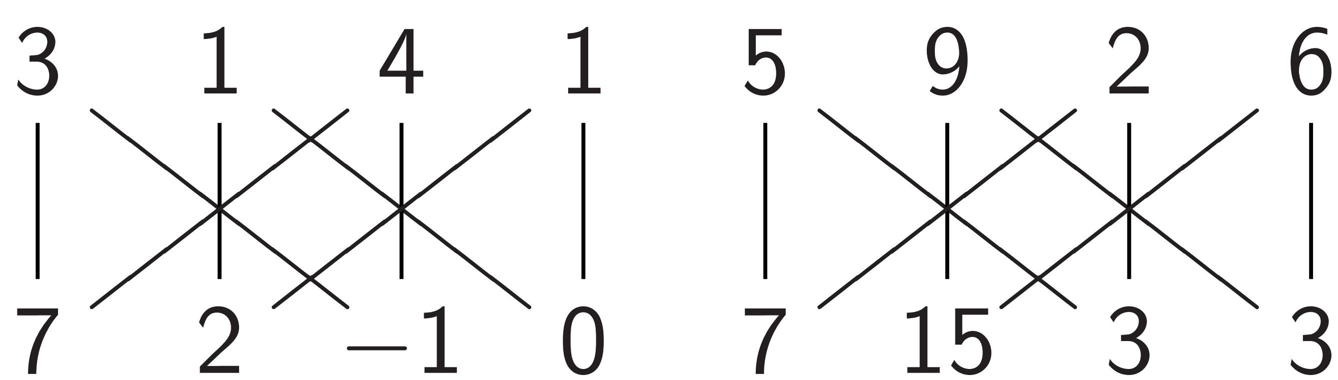
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 4. Hadamard$_2$:

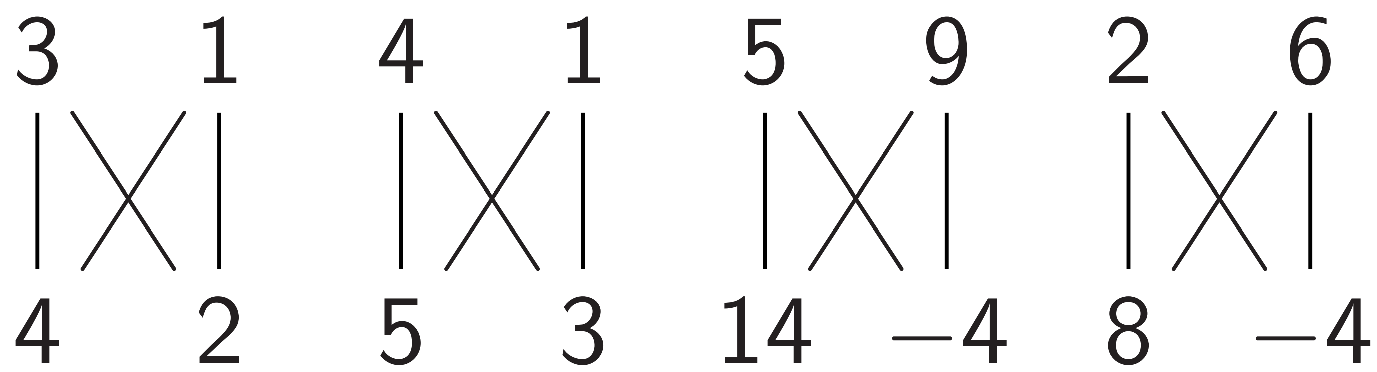
1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe.

Hadamard gates

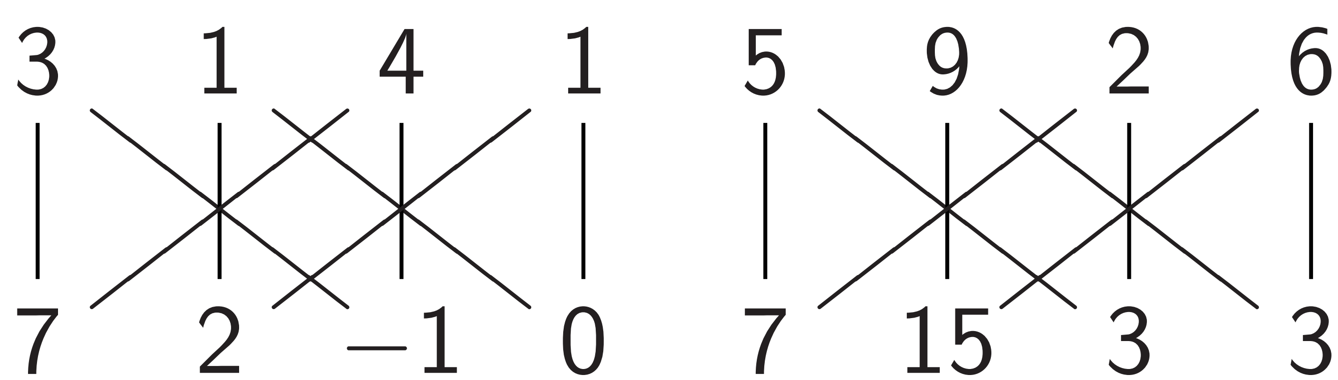
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5. CNOT$_{0,3}$:

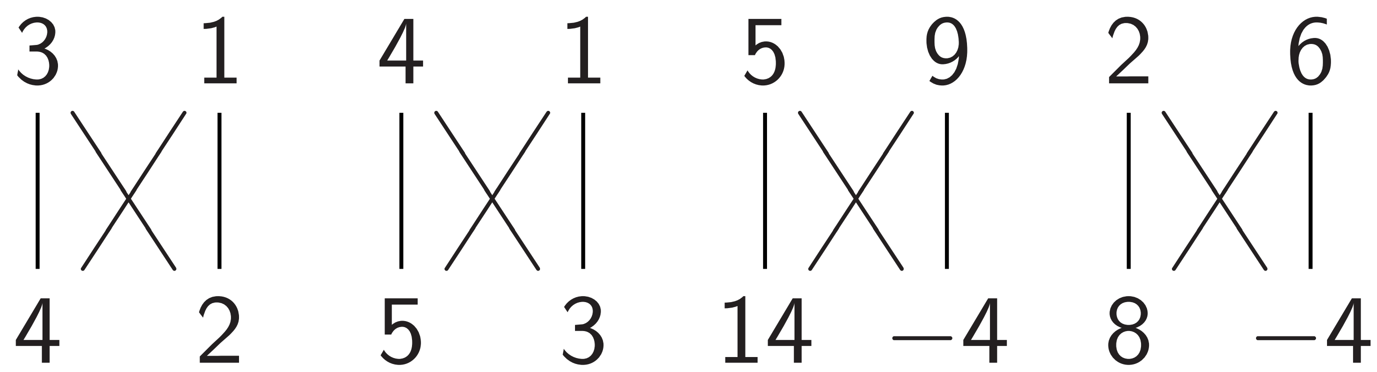
1, 0, 1, 0, 1, 0, 1, 0,
0, 1, 0, 1, 0, 1, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

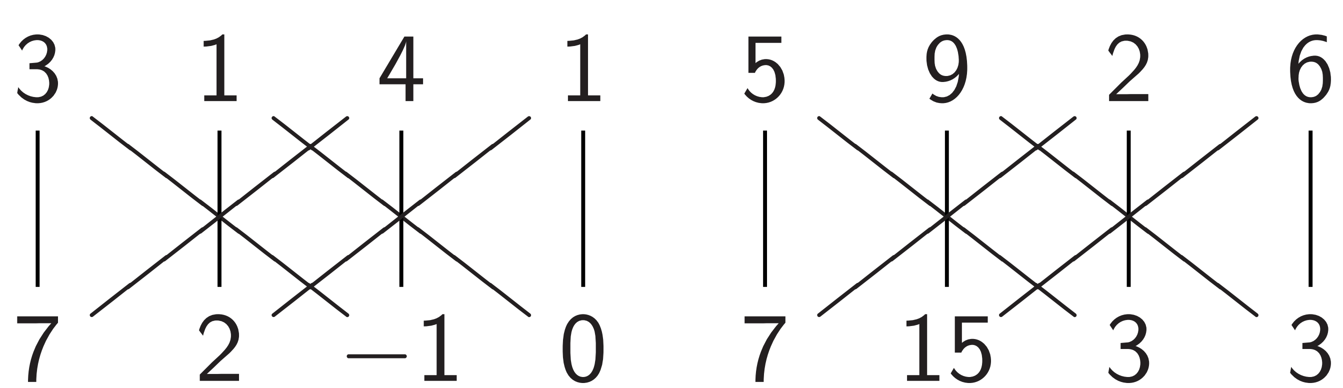
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5b. More shuffling:

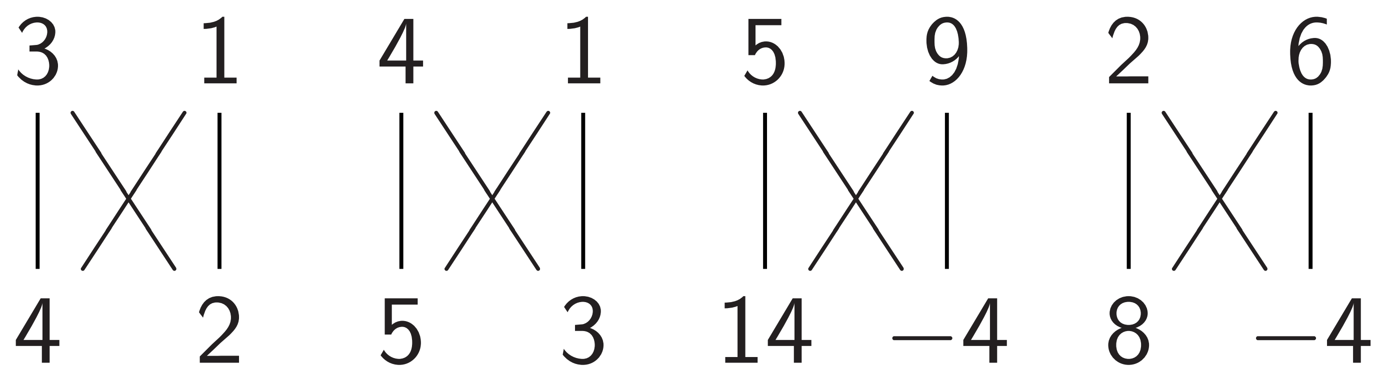
1, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

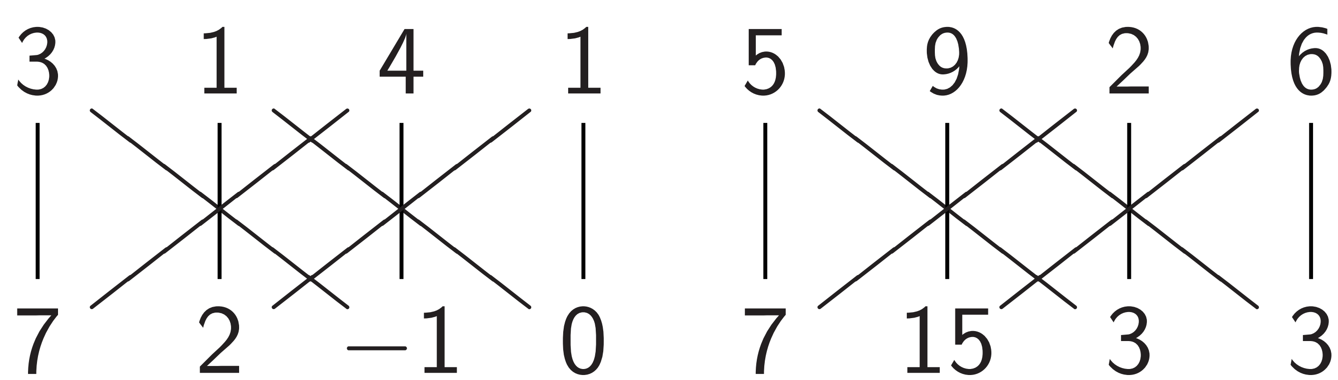
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5c. More shuffling:

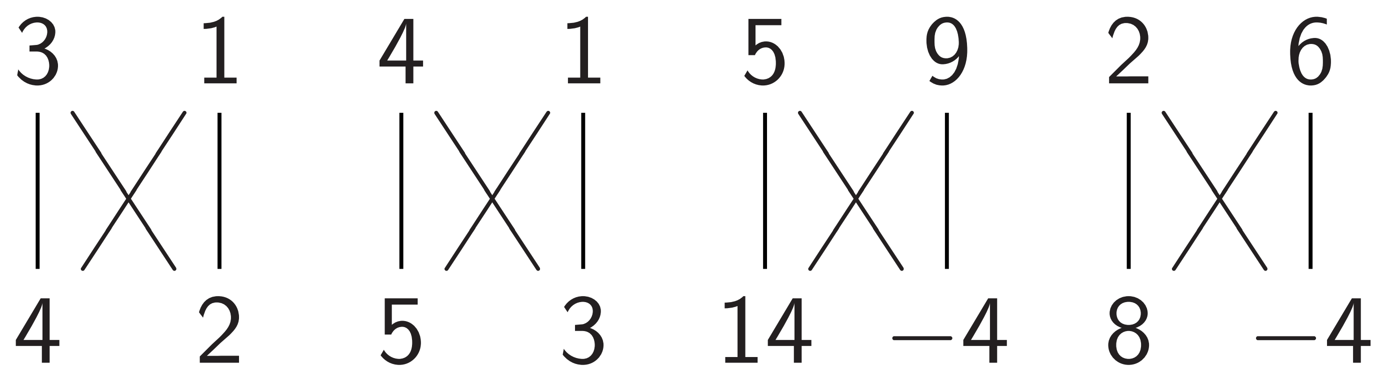
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Hadamard gates

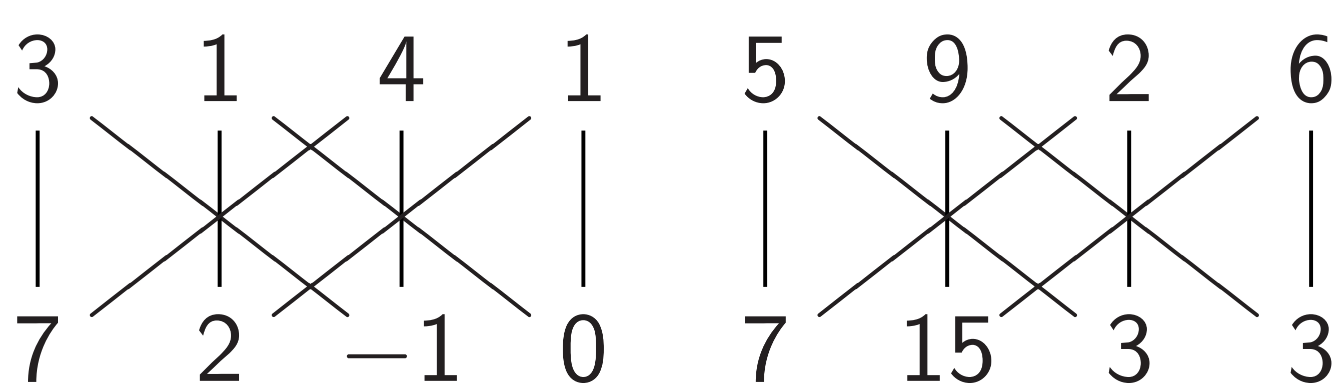
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5d. More shuffling:

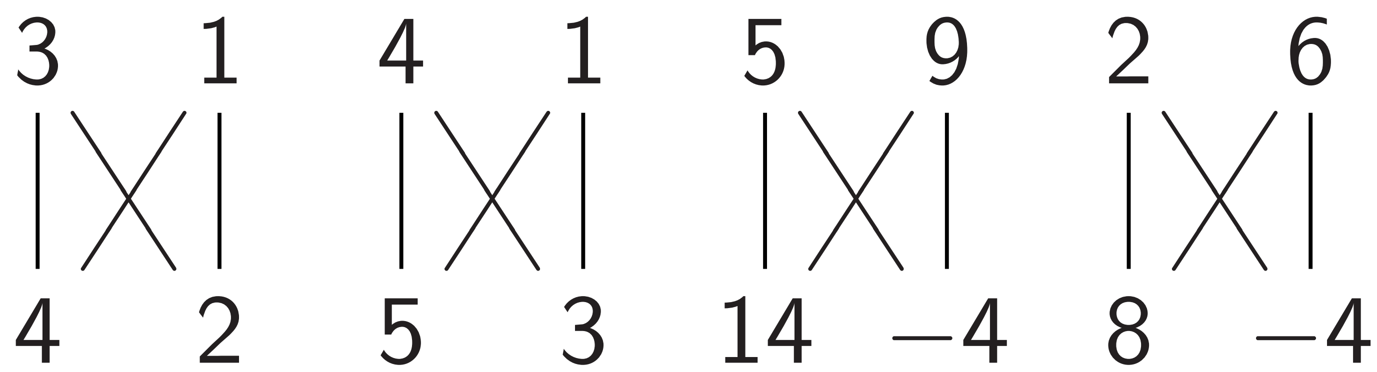
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

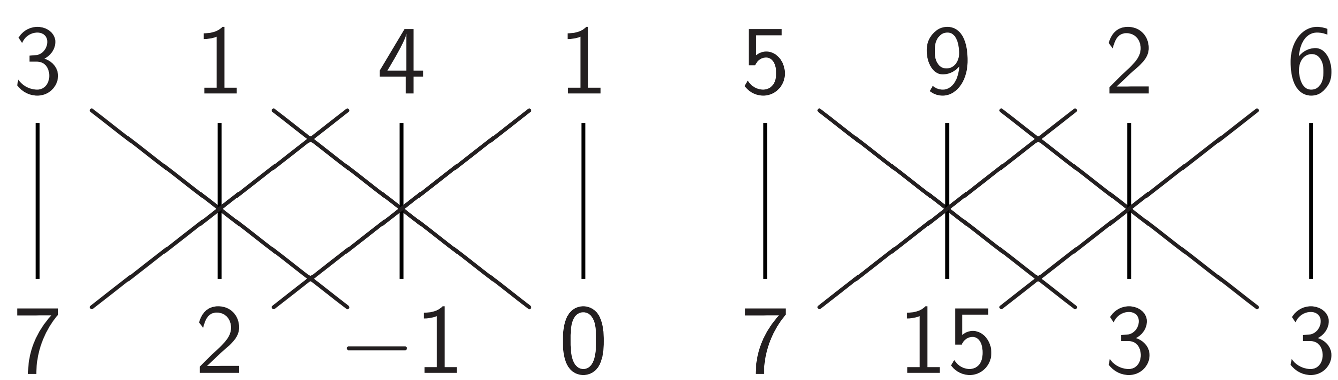
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5e. More shuffling:

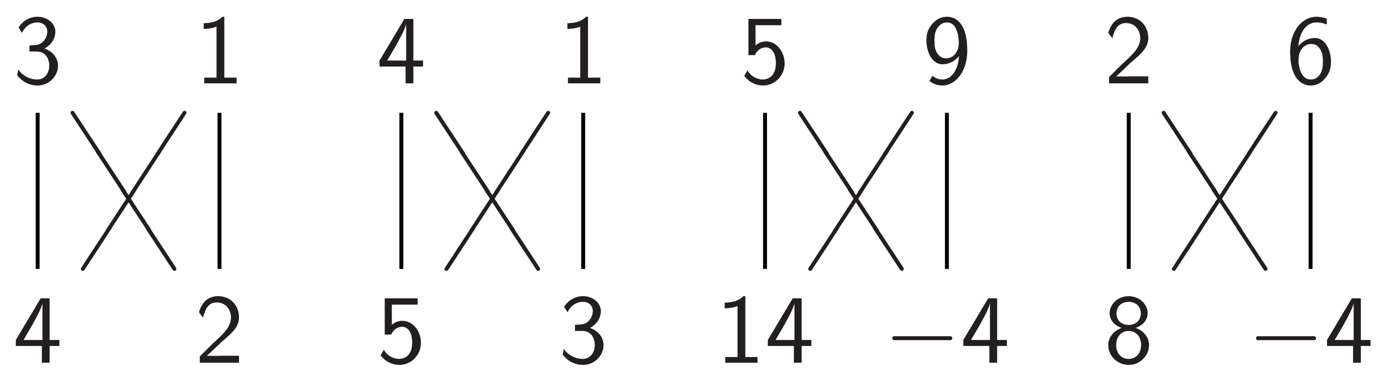
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

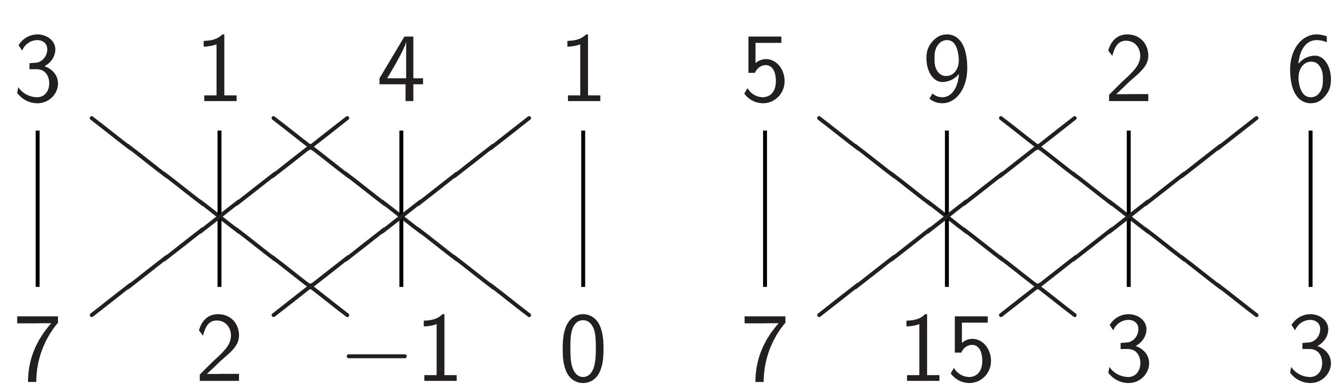
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5f. More shuffling:

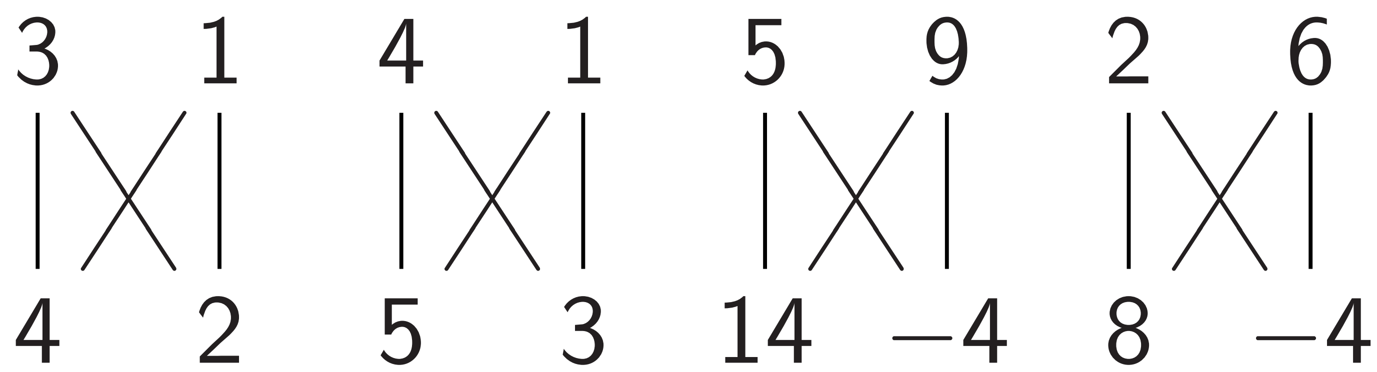
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

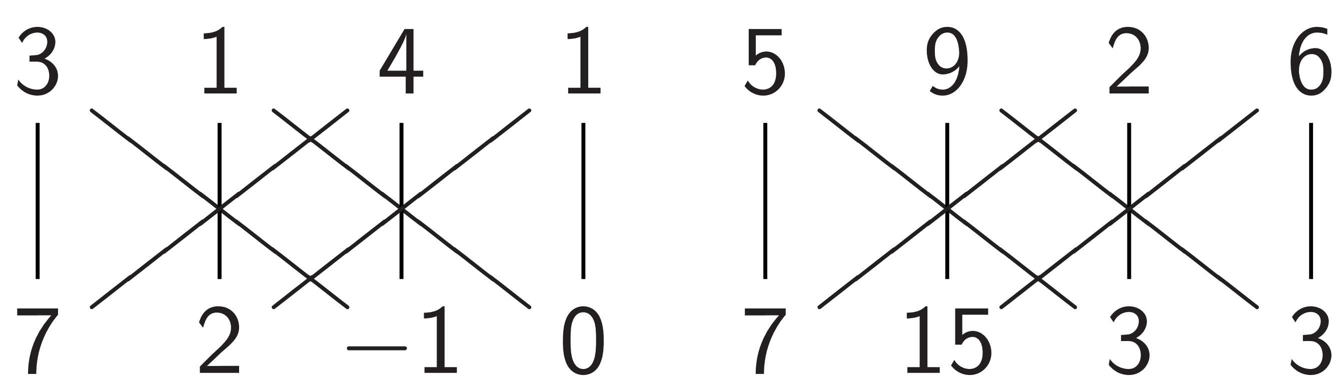
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5g. More shuffling:

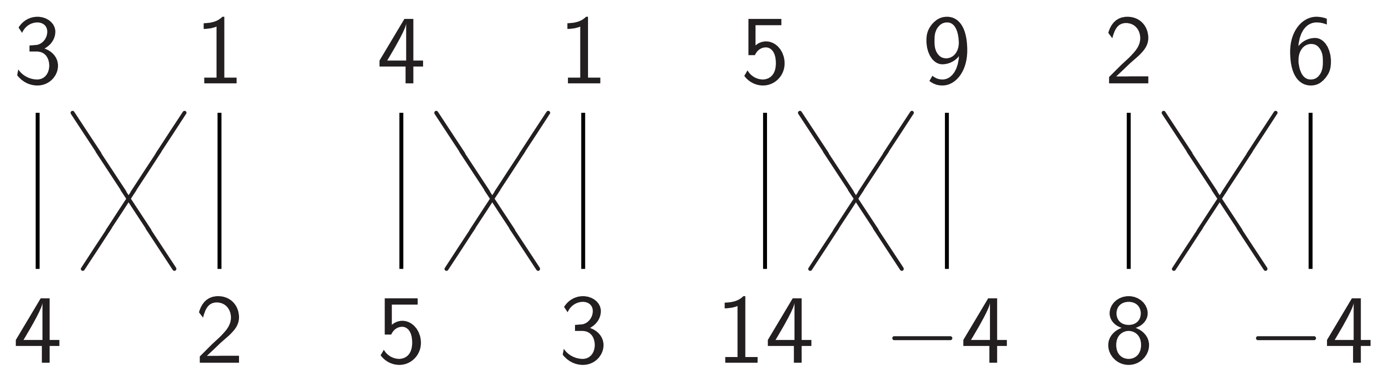
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Hadamard gates

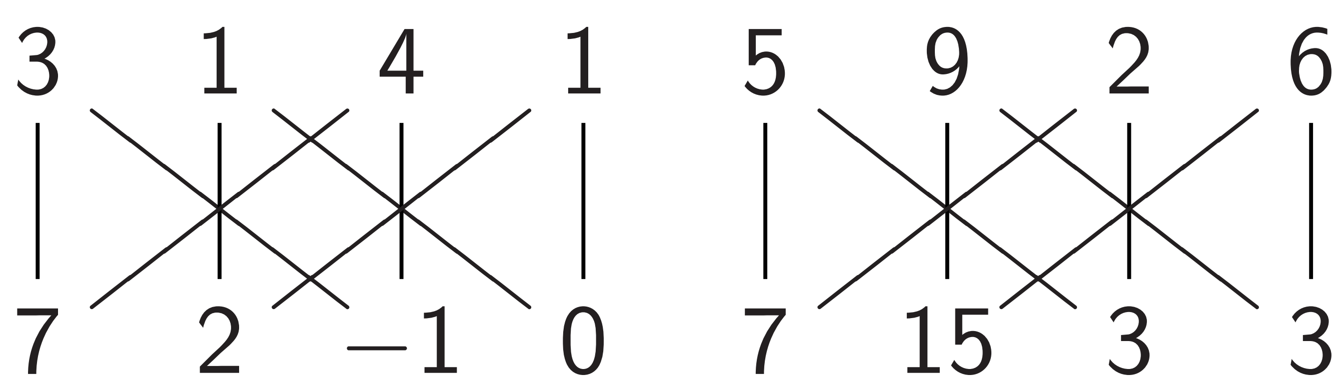
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5h. More shuffling:

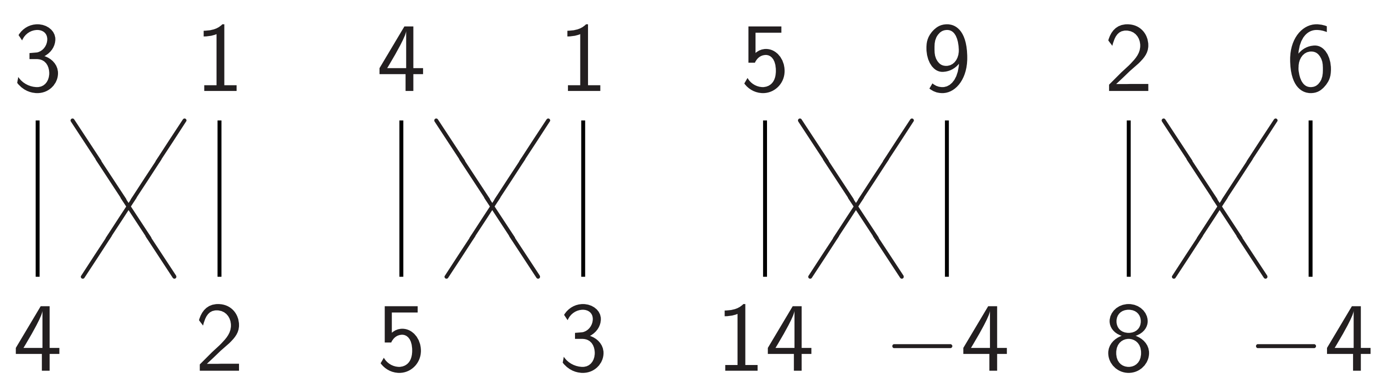
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

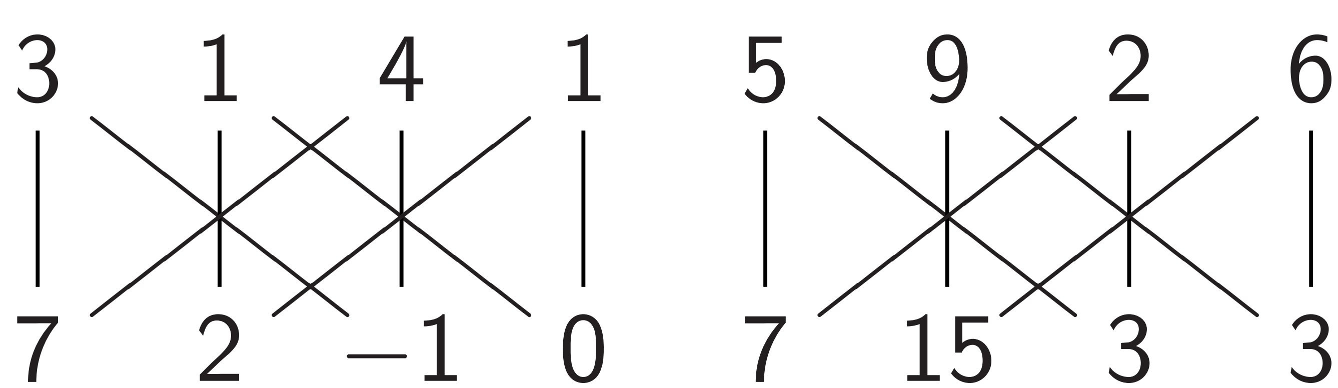
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5i. More shuffling:

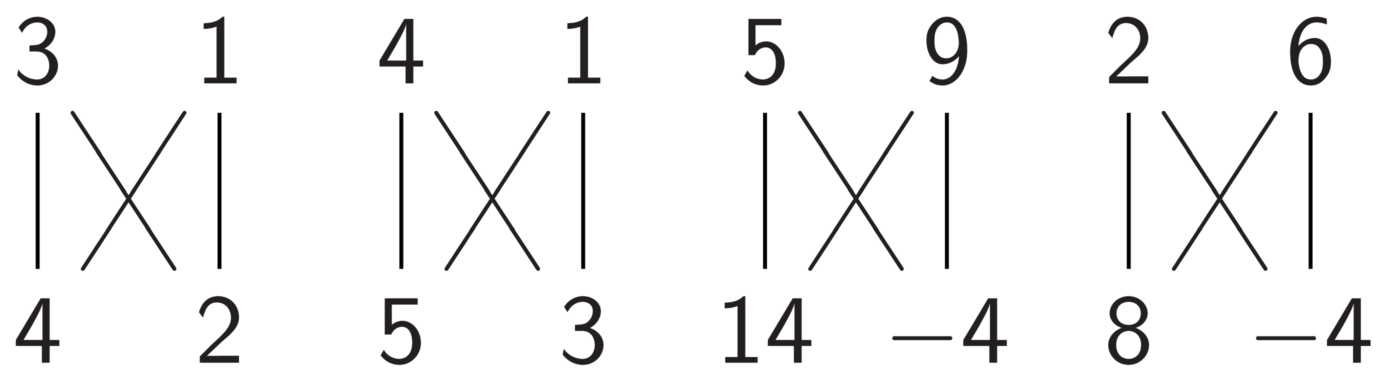
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

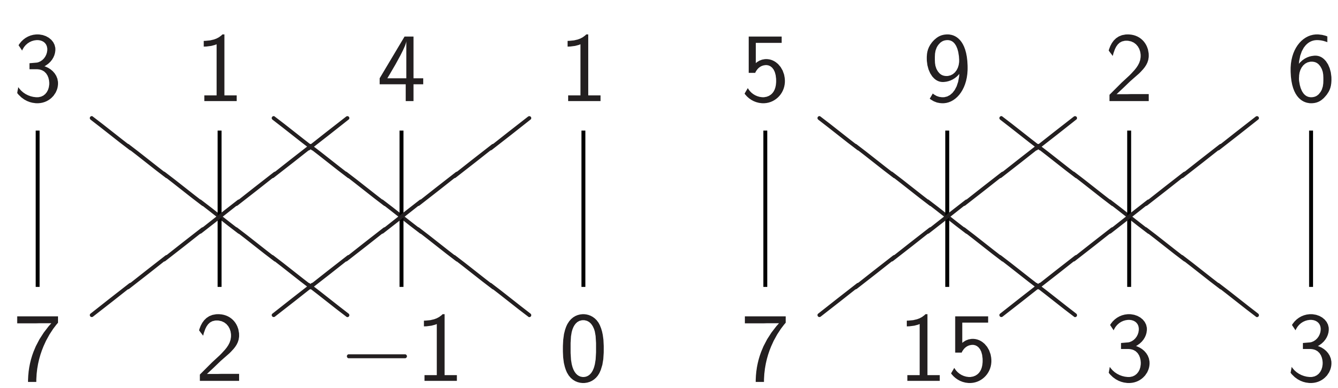
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5j. Final shuffling:

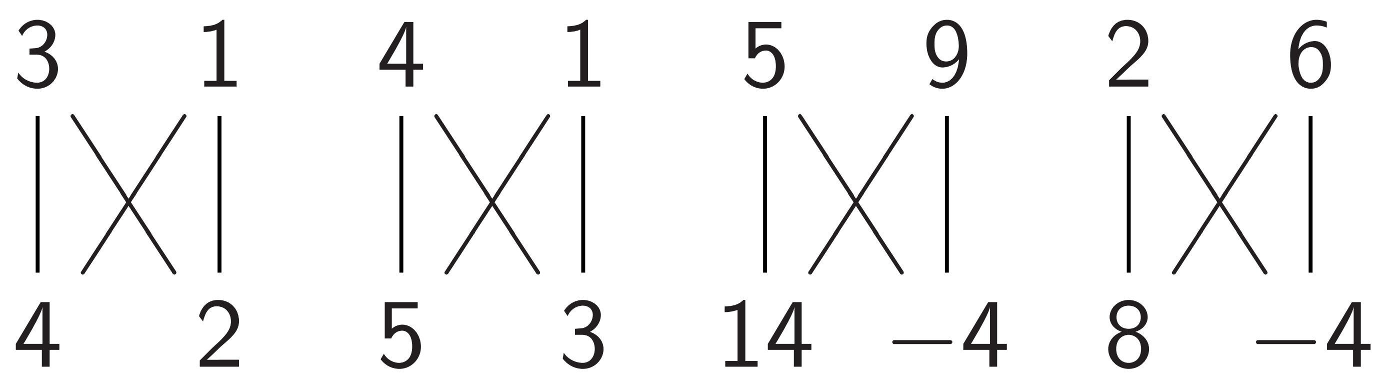
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.

Hadamard gates

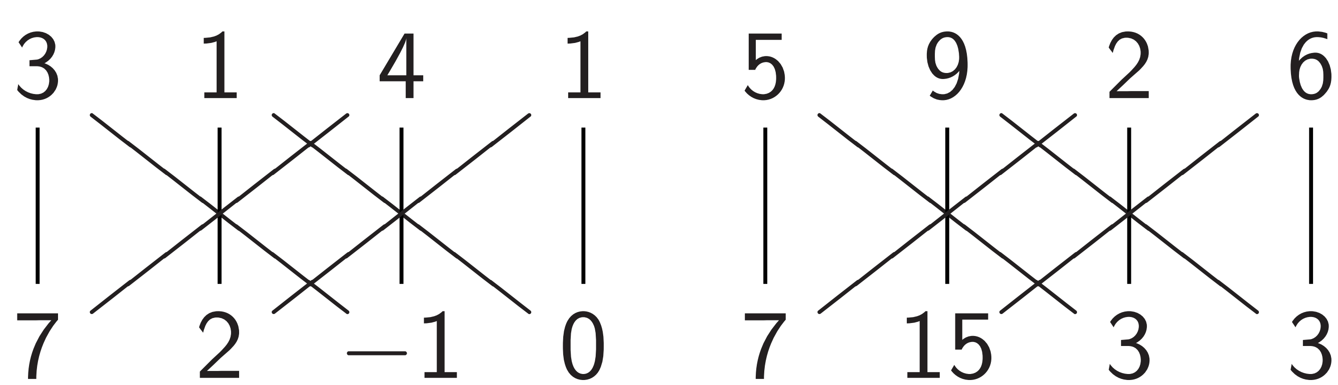
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 5j. Final shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

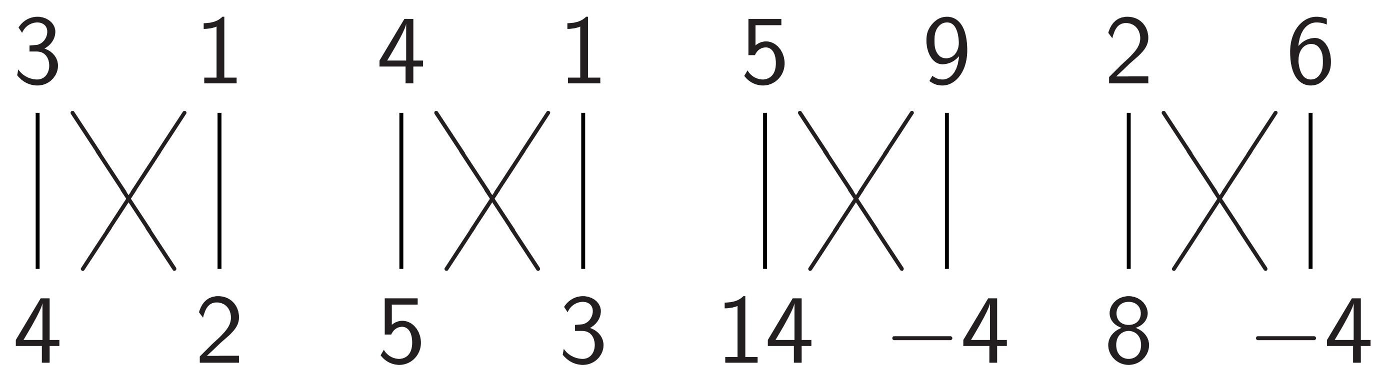
Each column is a parallel universe performing its own computations.

Surprise: u and $u \oplus 101$ match.

Hadamard gates

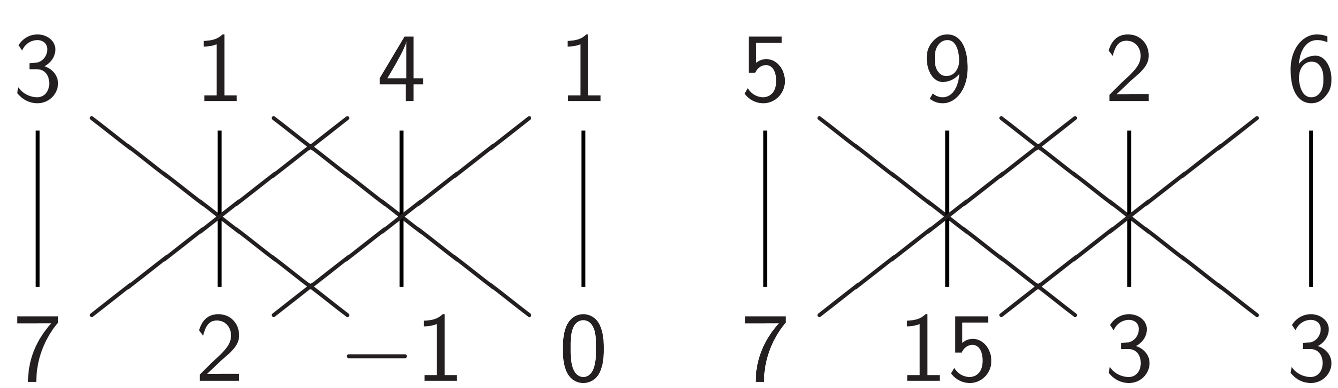
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

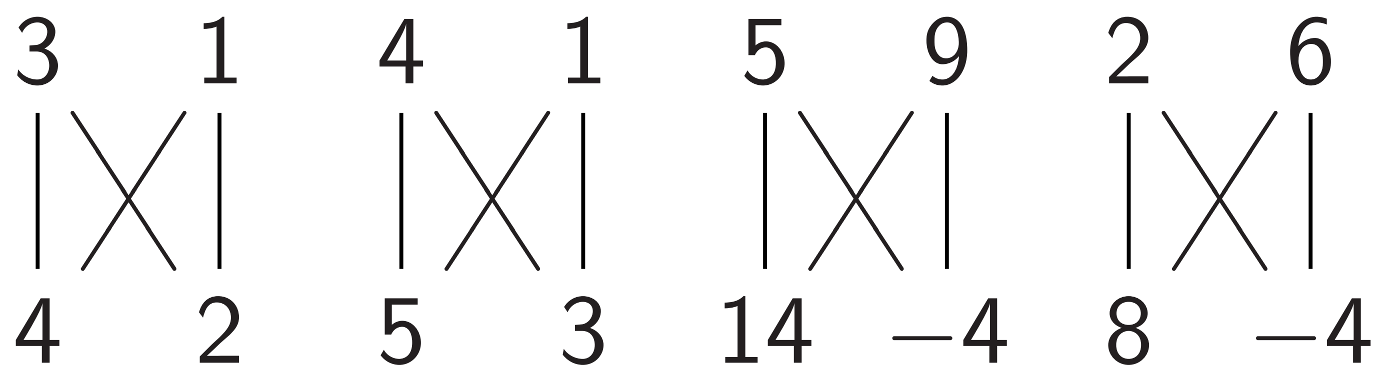
Step 6. Hadamard$_0$:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, $\overline{1}$, 0, 0, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 1, 0, 0, 1, $\overline{1}$,
1, $\overline{1}$, 0, 0, 1, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 0, 0, 1, $\overline{1}$, 0, 0.

Hadamard gates

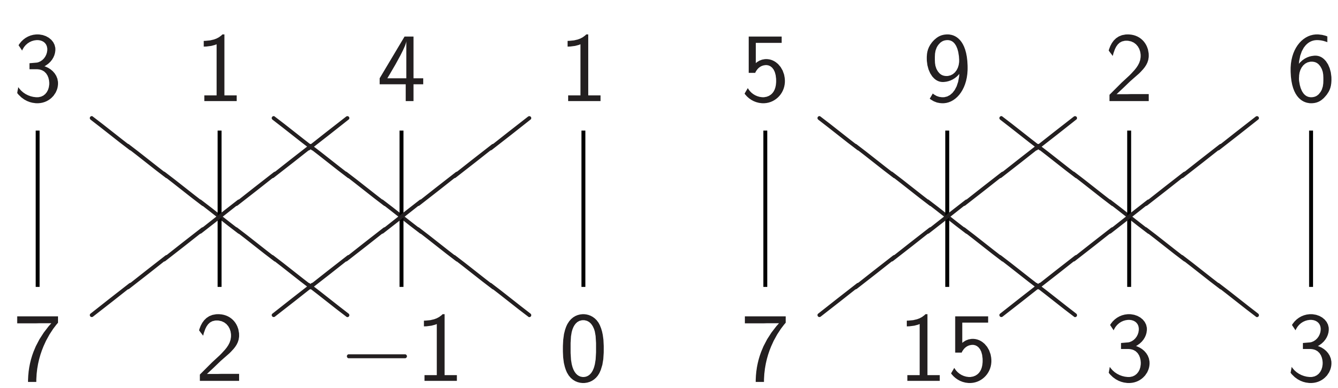
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

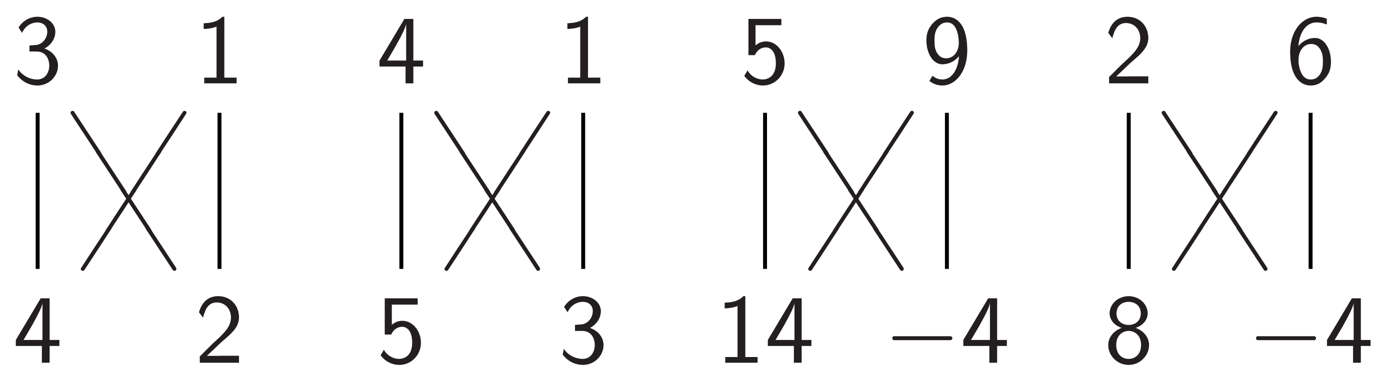
Step 7. Hadamard$_1$:

$0, 0, 0, 0, 0, 0, 0, 0,$
$1, \overline{1}, \overline{1}, 1, 1, 1, \overline{1}, \overline{1},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, \overline{1}, \overline{1}, 1, \overline{1}, \overline{1}, 1,$
$1, \overline{1}, 1, \overline{1}, 1, 1, 1, 1,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$1, 1, 1, 1, 1, \overline{1}, 1, \overline{1}.$

Hadamard gates

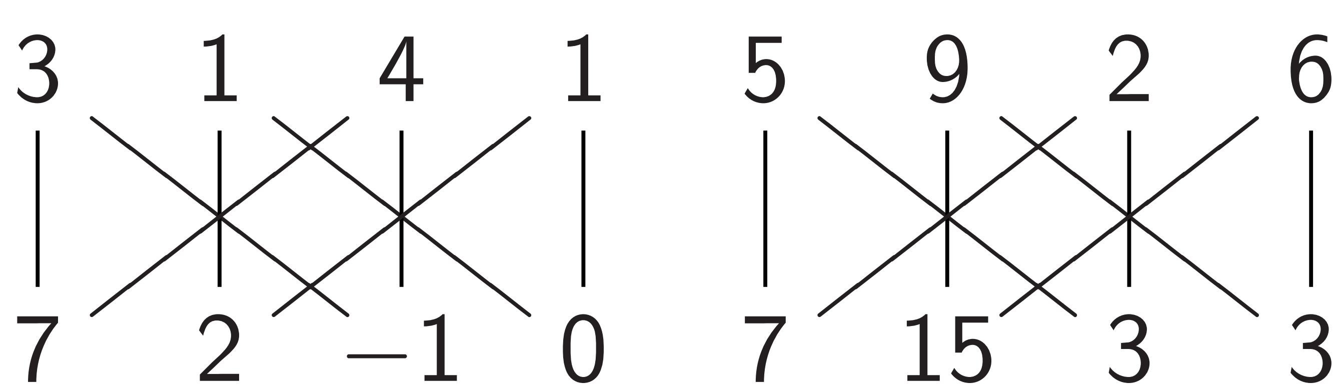
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 8. Hadamard$_2$:

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, 2, 0, \overline{2},$

$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2},$

$0, 0, 0, 0, 0, 0, 0, 0,$

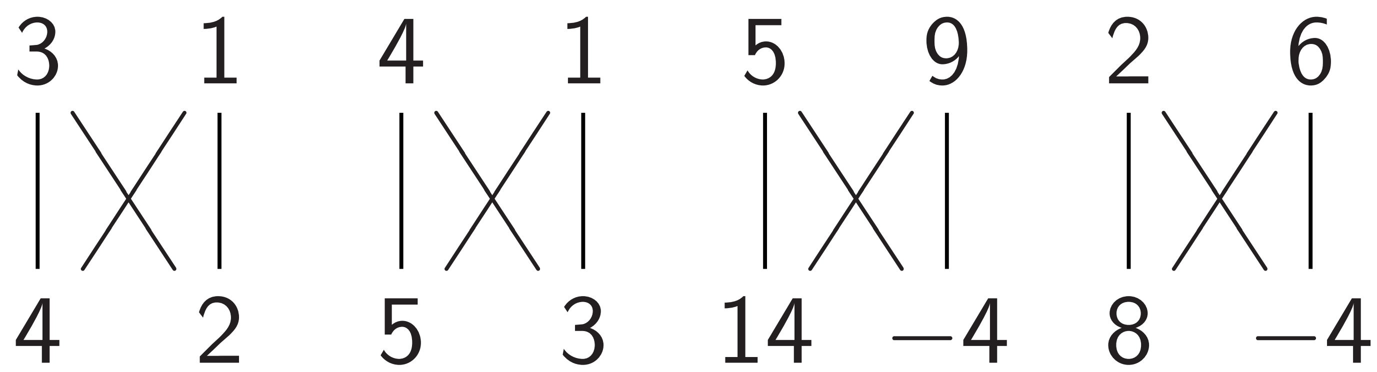
$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, 2, 0, 2.$

Hadamard gates

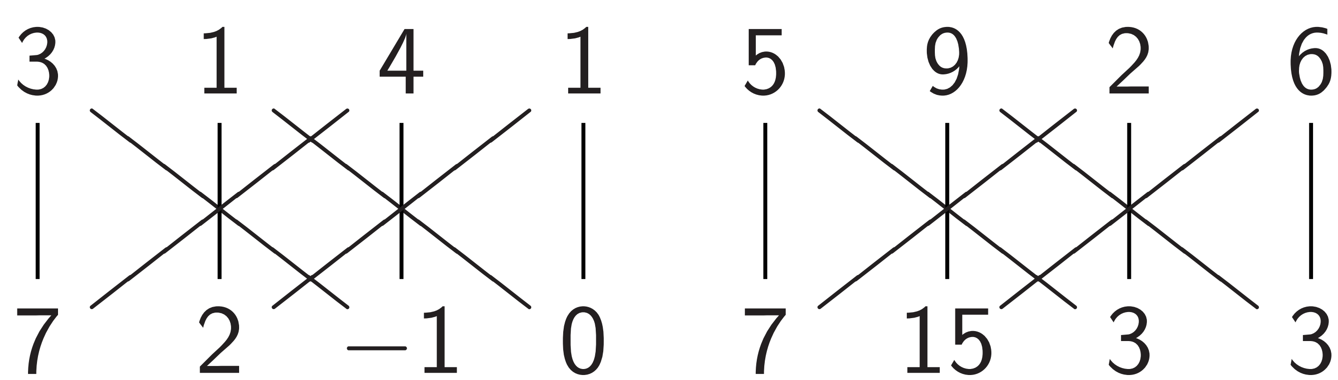
Hadamard$_0$:

$(a, b) \mapsto (a + b, a - b)$.



Hadamard$_1$:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.



Simon's algorithm

Step 8. Hadamard$_2$:

$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, \overline{2}, 0, 0, 2, 0, \overline{2},$
$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2},$
$0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0,$
$2, 0, 2, 0, 0, 2, 0, 2.$

Step 9: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.